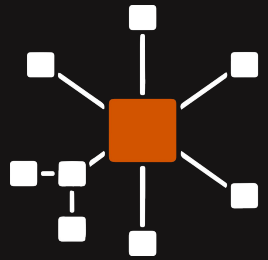

CSLib
The Lean Computer Science Library

Lean Together, 20 January 2026

Presenter:

Fabrizio Montesi
Director, Professor, FORM
CSLib Lead Maintainer

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University of Southern Denmark
Centre for Formal Methods and Future Computing
D·IAS
DANISH INSTITUTE FOR ADVANCED STUDY

Computing is under pressure

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More quantity

More systems!

More features!

Cheaper!

Faster!

Computing is under pressure

More quantity	More quality
More systems!	More secure!
More features!	More robust!
Cheaper!	More sustainable!
Faster!	More privacy!

A convergence of ambitions in 2025

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Clark Barrett

Stanford & Amazon

Swarat Chaudhuri

Google DeepMind & UT Austin

Jim Grundy

Amazon

Pushmeet Kohli

Google DeepMind

Fabrizio Montesi

FORM, University of Southern Denmark

Leonardo de Moura

Lean FRO & Amazon

A convergence of ambitions in 2025

➔ Scale up formal computer science and programming by building a common infrastructure. CSLib – www.cslib.io

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What is CSLib?

An open source repository (github.com/leanprover/cslib) of

1. computer science definitions and results,
2. verified software components, and
3. verification infrastructure.

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★ Written in Lean, with mathlib as dependency.
★ Curated by experts.
★ Done in collaboration with the Lean community and FRO.

Governance

- Steering Group (strategy, funding)
  - Clark Barrett, Swarat Chaudhuri, Jim Grundy, Pushmeet Kohli, Fabrizio Montesi, Leonardo de Moura.
- Maintainer Group (technical leadership and supervision)
  - Lead Maintainer: Fabrizio Montesi
  - Tech Leads: Alexandre Rademaker, Sorrachai Yingchareonthawornchai
  - Area Maintainers: Chris Henson, Kim Morrison
- Discussions on GitHub, Zulip, Email, …
  - https://github.com/leanprover/cslib/graphs/contributors

Who is CSLib for?

- Researchers
  - Verify your claims.
  - Speed up your development.
  - Consolidate and offer your findings through our APIs.

Who is CSLib for?

- Researchers
- Educators & Learners
  - Explore CS through a unified language.
  - Interact with the tool.

Who is CSLib for?

- Researchers
- Educators & Learners
- Programmers
  - Use CSLib components to write reliable software.
  - Use CSLib's languages and logics to model and implement systems.
  - Use CSLib's verification infrastructure to verify new and old code.

Who is CSLib for?

- Researchers
- Educators & Learners
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- AI Developers
  - Train AI on CSLib.
  - Increase the production of specs, programs, and proofs.

Who is CSLib for?

- Researchers
- Educators & Learners
- Programmers
- AI Developers
- Interdisciplinary Researchers
    - Explore the application of formal methods to provide actionable information to users.

Where are we now?

Credits:
https://github.com/leanprover/cslib/graphs/contributors

Current structure

```
Cslib/
    Algorithms/
    Computability/
    Foundations/  # General, reusable foundations
    Languages/    # Modelling and programming
    Logics/       # Various logics for reasoning
    [more to come...]
```

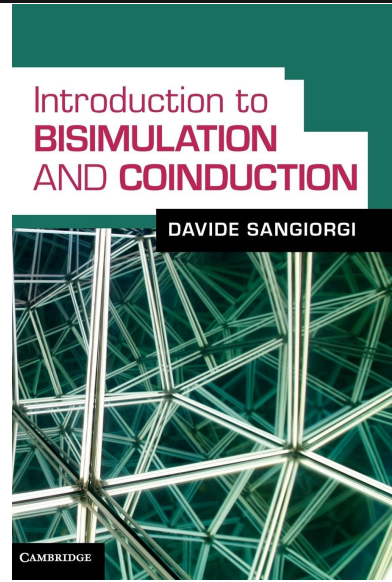
Current structure

Many answers found!

A lot more questions lie ahead…

Foundations

- ★ Labelled Transition Systems.
  - Classes of LTSs (image-finite, deterministic, etc.)
  - Bisimilarity, weak bisimilarity.
  - Similarity.
  - Trace equivalence (w/ inclusion in bisimilarity).
- ★ Reduction systems.
  - Lots about relations (confluence, etc.).
- ★ Freer monads.



Grind-first approach. (See Chris' talk tomorrow!)

Foundations

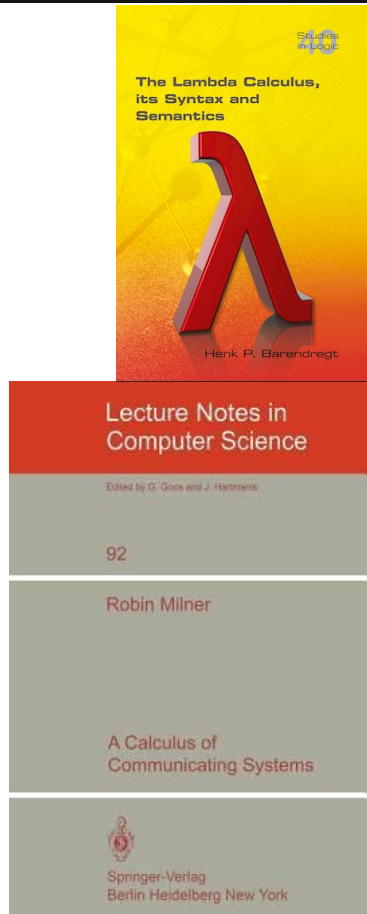
💡 Grind-first approach.

```
117  130    /-- The inverse of a bisimulation is a bisimulation. -/
118       - theorem Bisimulation.inv (h : Bisimulation lts r) :
119       -   Bisimulation lts (flip r) := by
120       -   simp only [Bisimulation] at h
121       -   simp only [Bisimulation]
122       -   intro s1 s2 hrinv μ
123       -   constructor
124       -   case left =>
125       -     intro s1' htr
126       -     specialize h s2 s1 hrinv μ
127       -     have h' := h.2 s1' htr
128       -     obtain ⟨ s2', h' ⟩ := h'
129       -     exists s2'
130       -   case right =>
131       -     intro s2' htr
132       -     specialize h s2 s1 hrinv μ
133       -     have h' := h.1 s2' htr
134       -     obtain ⟨ s1', h' ⟩ := h'
135       -     exists s1'
          131 + @[grind]
          132 + theorem Bisimulation.inv (h : lts.IsBisimulation r) :
          133 +   lts.IsBisimulation (flip r) := by grind [flip]
```

Languages

- ★ λ-calculus.
  - Untyped, typed (STLC, System Fsub).
  - Alternative formalisations of α-equivalence: named (standard), locally nameless, well-scoped (WIP).
- ★ Calculus of Communicating Systems (CCS).
  - Behavioural theory, w/ equivalences and proof that bisimilarity is a congruence.
- ★ Combinatory Logic (SKI).
  - Rice's theorem.

💡 Generality; reusable APIs.



Languages

💡 Generality; reusable APIs.

```
/-- A type `α` has a computable `fresh` function if it is always possible, for any finite set
of `α`, to compute a fresh element not in the set. -/
class HasFresh (α : Type u) where
  /-- Given a finite set, returns an element not in the set. -/
  fresh : Finset α → α
  /-- Proof that `fresh` returns a fresh element for its input set. -/
  fresh_notMem (s : Finset α) : fresh s ∉ s
```

Logics

- ★ Linear Logic.
  - ○ Sequent calculus.
  - ○ Many logical equivalences.
    - ■ $a \otimes 0 \equiv 0$, $a \otimes (b \oplus c) \equiv (a \otimes b) \oplus (a \otimes c)$, etc.
  - ○ η-expansion (proven correct).
  - ○ Phase semantics.
- ★ WIP: Propositional Logic.

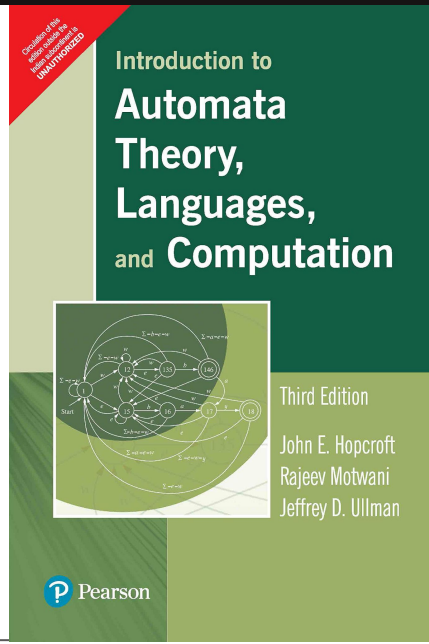
💡 Derivations are in Type; notation can be controversial; ‘pray to grind’ for dealing with Multiset rewriting.

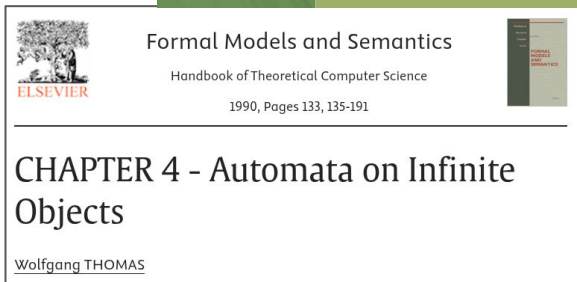
Logics

Automata and Their Languages

- ★ Deterministic and nondeterministic automata based on labelled transition systems.
- ★ Acceptors for finite and infinite words (generalisations of DFA, NFA, εNFA, Buchi, Muller, etc.).
- ★ Theory of ω-languages and ω-regular languages.

💡 Interactions between Labelled Transition Systems and Automata.





Automata and Their Languages

Interactions between Labelled Transition Systems and Automata.

Automata and Their Languages

Interactions between Labelled Transition Systems and Automata.

```
/-- A nondeterministic automaton extends an `LTS` with a set of initial states. -/
structure NA (State Symbol : Type*) extends LTS State Symbol where
  /-- The set of initial states of the automaton. -/
  start : Set State
```

Automata and Their Languages

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```
/-- A nondeterministic automaton that accepts finite strings (lists of symbols). -/
structure FinAcc (State Symbol : Type*) extends NA State Symbol where
  /-- The set of accepting states. -/
  accept : Set State
```

Automata and Their Languages

Interactions between Labelled Transition Systems and Automata.

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structure FinAcc (State Symbol : Type*) extends NA State Symbol where
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  accept : Set State
```

```
/-- Nondeterministic Muller automaton. -/
structure Muller (State Symbol : Type*) extends NA State Symbol where
  /-- The set of sets of accepting states. -/
  accept : Set (Set State)
```

Automata and Their Languages

Interactions between Labelled Transition Systems and Automata.

LTS images:

```
/-- The `DA.FinAcc` constructed from an `NA.FinAcc` has the same language. -/
@[scoped grind _=_]
theorem toDAFinAcc_language_eq {na : NA.FinAcc State Symbol} :
    language na.toDAFinAcc = language na := by
  ext xs
  grind
```

Automata and Their Languages

Interactions between Labelled Transition Systems and Automata.

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Saturation:

```
theorem toNAFinAcc_language_eq {ena : εNA.FinAcc State Symbol} :
    language ena.toNAFinAcc = language ena := by
  ext xs
  have : ∀ s s', ena.saturate.MTr s (xs.map some) s' = ena.saturate.noε.MTr s xs s' := by
    simp [LTS.noε_saturate_mTr]
  grind
```

Automata and Their Languages

💡 Interactions between Labelled Transition Systems and Automata.

LTS images:

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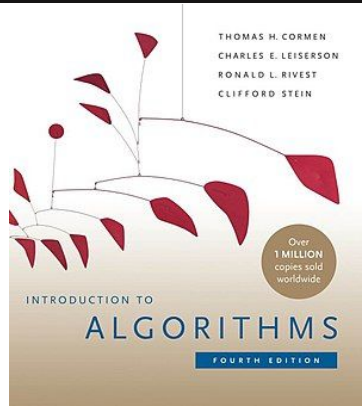
```

Plus general results on infinite executions and saturation (not shown).

Algorithms

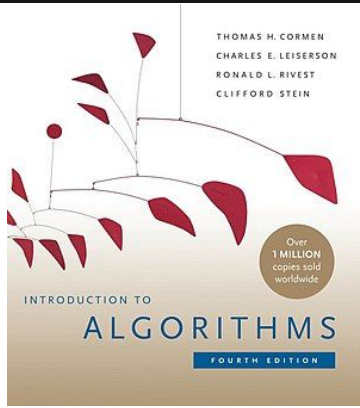


- ★ Framework for writing algorithms and proving their time complexity in terms of atomic operations.
  - MergeSort (functional correctness, time complexity on number of comparisons).
  - WIP: more algorithms, space complexity, more automation.
- ★ WIP: Framework based on query models.
  - Separate definition of query model.
  - Algorithms perform queries.

💡 Balancing robustness with ergonomics.

Algorithms

Balancing robustness with ergonomics.

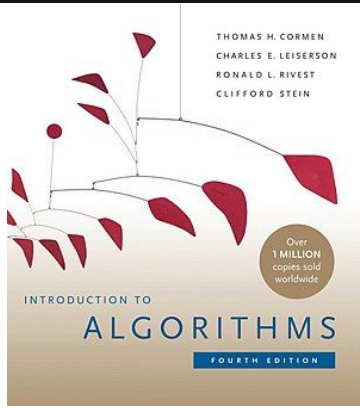



```
def merge :  List α → List α → TimeM (List α)
  | [], ys => return ys
  | xs, [] => return xs
  | x::xs', y::ys' => do
    let c ← ✓ (x ≤ y : Bool)
    if c then
      let rest ← merge xs' (y::ys')
      return (x :: rest)
    else
      let rest ← merge (x::xs') ys'
      return (y :: rest)
```

```
/-- Time complexity of mergeSort -/
theorem mergeSort_time (xs : List α) :
    let n := xs.length
    (mergeSort xs).time ≤ n * clog2 n:= by
  grind [mergeSort_time_le, timeMergeSortRec_le]
```

Algorithms

Balancing robustness with ergonomics.

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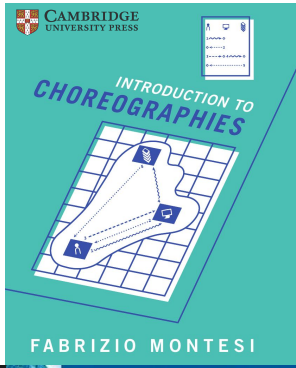
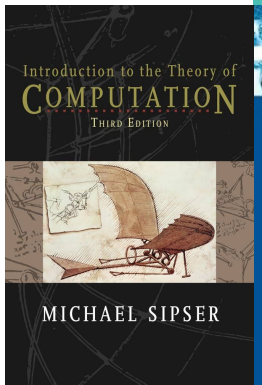
```

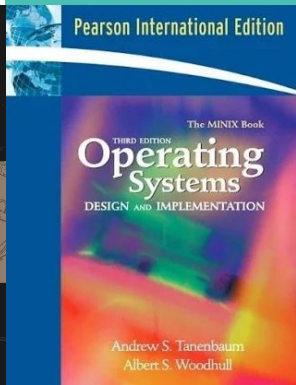
What next?

Next steps

- ➔ More Computer Science.
  - Database theory.
  - Cost|Probabilistic|Denotational|* semantics.
  - Concurrency theory (Petri nets, pi-calculus, etc.).
  - Choreographic languages (security protocols, choreographic programming, business processes, multiparty session types, etc.).
  - Logics: modal logics, separation logic, etc.
  - …





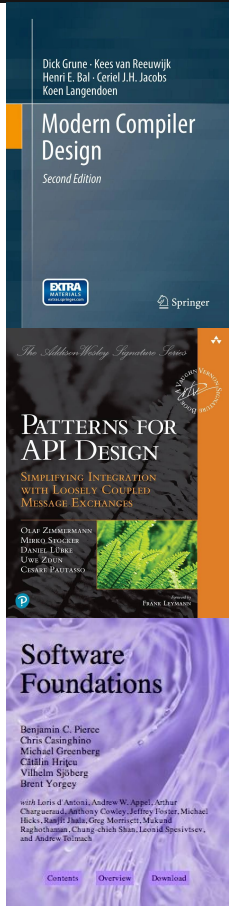


Next steps

- ➔ More Computer Science.
- ➔ Verification infrastructure.
  - ◆ Development of IRs for verification (Boole).
  - ◆ Compilers from mainstream languages to Boole.
  - ◆ Integration of logics with verification infrastructure.

Next steps

- ➔ More Computer Science.
- ➔ Verification infrastructure.
- ➔ Programming infrastructure.
  - ◆ Design and implementation of high-level languages with certified compilers.
  - ◆ Bridge to SE: Architecture Description Languages, Design Patterns, Provably-Correct Refactorings, etc.
  - ◆ Integration w/ system middleware (microservices, serverless, etc.).



Thank you for the amazing contributions!

Thank you for listening!

And thanks for funding my time and our group to:
:-)

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