Ozone: Fully Out-of-Order Choreographies

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Abstract

Choreographic programming is a paradigm for writing distributed applications. It allows programmers to write a single program, called a choreography, that can be compiled to generate correct implementations of each process in the application. Although choreographies provide good static guarantees, they can exhibit high latency when messages or processes are delayed. This is because processes in a choreography typically execute in a fixed, deterministic order, and cannot adapt to the order that messages arrive at runtime. In non-choreographic code, programmers can address this problem by allowing processes to execute out of order—for instance by using futures or reactive programming. However, in choreographic code, out-of-order process execution can lead to serious and subtle bugs, called *communication integrity violations (CIVs)*.

In this paper, we develop a model of choreographic programming for out-of-order processes that guarantees absence of CIVs and deadlocks. As an application of our approach, we also introduce an API for safe non-blocking communication via futures in the choreographic programming language Choral. The API allows processes to execute out of order, participate in multiple choreographies concurrently, and to handle unordered or dropped messages as in the UDP transport protocol. We provide an illustrative evaluation of our API, showing that out-of-order execution can reduce latency by overlapping communication with computation.

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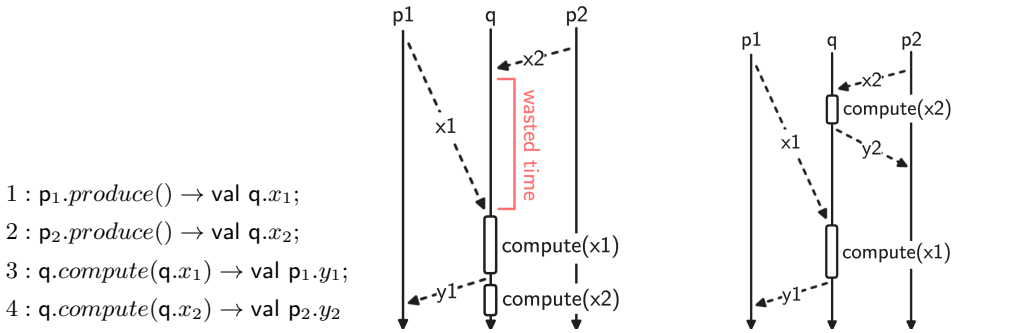
1 Introduction

Choreographic programming [16] is a paradigm that simplifies writing distributed applications. In contrast to a traditional development style, where one implements a separate program for each type of process in the system, choreographic programming allows a programmer to define the behaviors of all processes together in a single program called a *choreography* [17]. Through *endpoint projection (EPP)*, a choreography can be compiled to generate the programs implementing each process that would otherwise need to be written by hand. Aside from convenience, the advantage of this approach is that certain classes of bugs (such as deadlocks) are impossible *by construction* [5]. Choreographic programming has been applied to popular languages such as Java [9] and Haskell [22], and has been used to implement real-world protocols such as IRC [14].

Processes in choreographic programs typically execute in a fixed, sequential order. Consider Figure 1a, which shows a simple choreography performed by processes p_1, p_2, and q. The syntax $\mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x$ means "p evaluates expression e and sends the result to q, which binds the result to a local variable x". According to the usual semantics for choreographies, $\mathsf{p}_1.\mathit{produce}()$ and $\mathsf{p}_2.\mathit{produce}()$ in the choreography can be evaluated in parallel because p_1 and p_2 are distinct processes [17]. However, q must execute each step sequentially: first q



(a) Choreography **(b)** In-order execution **(c)** Out-of-order execution

Figure 1 A choreography where out-of-order execution can improve performance.

waits until it receives x_1; then q waits until it receives x_2; and only then can q send p_1 the result of processing x_1.

Figure 1b depicts an execution of the choreography, showing the drawback of a fixed processing order: if x_2 arrives before x_1, q wastes time waiting for x_1 instead of processing x_2. Ideally, q would evaluate $\mathit{compute}(\mathsf{q}.x_1)$ and $\mathit{compute}(\mathsf{q}.x_2)$ according to the arrival order of x_1 and x_2, as shown in Figure 1c. Assuming these two expressions are safe to reorder, such an optimization would allow q to overlap computation with communication and reduce the average latency experienced by p_1 and p_2. We are therefore interested in studying choreographic programming models where processes may execute some statements out-of-order, or even concurrently. We call such processes *out-of-order processes* and the corresponding choreographies *(fully) out-of-order choreographies*.

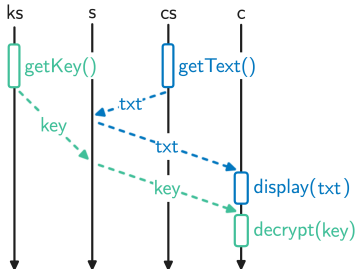
Processes with out-of-order features have been considered in prior work. Process models such as the actor model [1] or the π-calculus with delayed receive [15] are expressive enough to implement the behavior in Figure 1c, but these models lack the static guarantees of choreographic programming. More recently, Montesi gave a semantics for *nondeterministic choreographies* [17], i.e., choreographies with nondeterministic choice. Nondeterministic choreographies can implement the execution in Figure 1c, but they are unwieldy when it comes to expressing out-of-order process execution: they require explicitly writing all possible schedulings. For our example, we would get a choreography twice the size of the one in Figure 1a. Consequently, nondeterministic choreographies are both hard to write and brittle—a typical drawback when using syntactic operators to express interleavings. This raises the question:

> *Can we develop a choreographic programming model for out-of-order processes that marries the simple syntax of Figure 1a with the semantics of Figure 1c?*

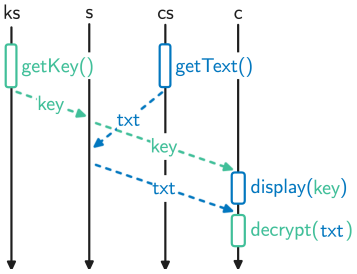
The simplicity of this problem is deceptive, since common-sense approaches can lead to pernicious compiler bugs. For instance, consider Figure 2: two microservices cs (a "content service") and ks (a "key service") send values $\mathit{txt}, \mathit{key}$ to a server s (lines 1 and 2). The server in turn forwards those values to a client c (lines 3 and 4). Notice that if s is an out-of-order process, then it can forward the results in any order, as shown in Figures 2b and 2c. This causes a problem for c: since both txt and key were sent by s, and since both values have the same type (String), c has no way to determine whether the first message contains txt (as in Figure 2b) or key (as in Figure 2c). This problem is easy for compiler writers to miss, leading to disastrous nondeterministic bugs where variables are bound to the wrong values. We call such bugs *communication integrity violations (CIVs)*.

1 : cs.$getText()$ → val s.txt;
2 : ks.$getKey()$ → val s.key;
3 : s.txt → val c.txt;
4 : s.key → val c.key;
5 : c.$display(txt)$;
6 : c.$decrypt(key)$

(a) Choreography



(b) Safe execution



(c) Buggy execution

Figure 2 A choreography where naïve out-of-order execution is unsafe. Process c cannot distinguish whether the first message it receives represents *key* or *txt*.

In this paper, we investigate CIVs and other complications that arise from mixing choreographies with out-of-order processes. Although the problem in Figure 2 can easily be solved by attaching static information (such as variable names) to each message, we show in Section 2 that a general solution requires mixing static and dynamic information, replicated across multiple processes. We also find that formalizing fully out-of-order choreographies requires several features uncommon in standard choreographic programming models, such as scoped variables and an expanded notion of well-formedness.

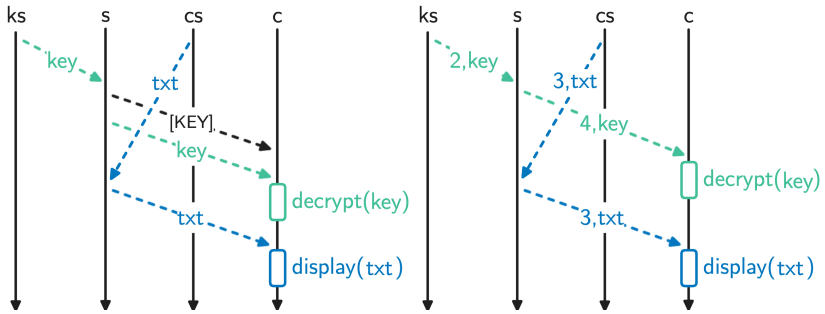
We make the following key contributions:

1. We present O_3, a formal model for asynchronous, fully out-of-order choreographies. Our model prevents CIVs by attaching *integrity keys* to messages. A nice consequence of our solution is that messages no longer need to be delivered in FIFO order. We prove that O_3 choreographies ensure deadlock-freedom (Theorem 2) and communication integrity (Theorem 4).
2. We present an EPP algorithm to project O_3 choreographies into out-of-order processes. We prove an operational correspondence theorem, which states that a choreography and its projection evolve in lock-step (Theorem 6). The key to making this proof tractable is a new notion of well-formedness that formalizes a communication integrity invariant. The theorem implies that a correct compiler will not generate code with deadlocks or CIVs.
3. As an application of our approach, we present a non-blocking communication API called *Ozone* for the choreographic programming language Choral [9]. Choreographies implemented with Ozone can use futures [2] to process messages concurrently (as in Figure 1c) without violating communication integrity. Ozone also allows programmers to handle dropped or unordered messages, as might occur in the UDP transport protocol. We give a proof-of-concept evaluation for Ozone with two microbenchmarks, confirming that out-of-order execution can indeed reduce latency for certain operations.

The outline of the paper is as follows. Section 2 explores CIVs and other issues in out-of-order choreography models. Section 3 presents our formal model O_3. Section 4 presents our model for out-of-order processes and our EPP algorithm. Section 5 presents our non-blocking API for Choral and our evaluation. We conclude with related work in Section 6 and discussion in Section 7.

2 Overview

In this section we explore the challenges that must be solved to develop a fully out-of-order choreography model, along with our approach.



(a) Using selections **(b)** Using integrity keys

Figure 3 Two approaches to prevent CIVs: selections and integrity keys.

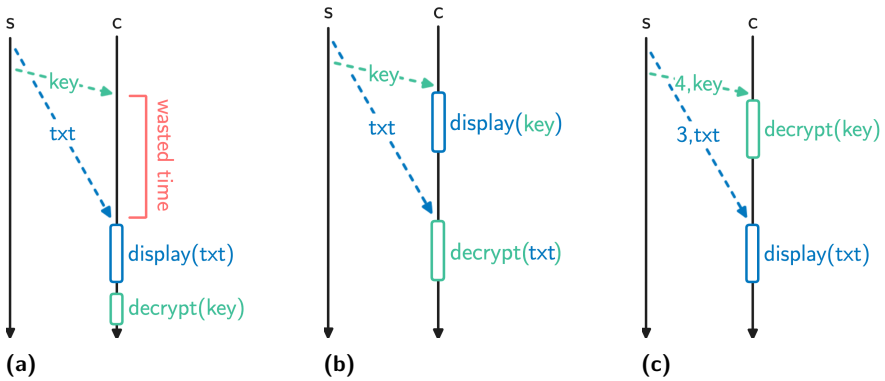


Figure 4 The challenges of non-FIFO delivery. Part (a) depicts head-of-line blocking when using a FIFO transport protocol: The message containing k arrives first, but it cannot be processed until t arrives. Part (b) depicts a CIV caused by using an unordered transport protocol without integrity keys. Part (c) depicts how the processes can use integrity keys to prevent CIVs.

2.1 Intraprocedural Integrity

Informally, communication integrity is the property that messages communicated in a choreography are bound to the correct variables. To ensure this property, processes sometimes need extra information; in Figure 2, process c needs to know which value will arrive first: *txt* or *key*.

A traditional solution would be for s to send a *selection* to c. Selections [17] are communications of constant values, used in choreography languages when one process makes a control flow decision that other processes must follow. Figure 3a shows how s could send the selection [KEY] to inform c that *key* will arrive before *txt*. Indeed, this is the approach used by nondeterministic choreographies [17]. However, selections impose overhead: any time nondeterminism could occur, the programmer would need to insert new selection messages. These extra messages would have both a cognitive cost for the programmer (as programs become littered with selections) and a runtime cost in the form of an extra message.

Instead, we opt to pair each message with a disambiguating tag called an *integrity key*. When c receives a message, it checks the integrity key to find the meaning of the message. Figure 3b uses *line numbers* as integrity keys: for example, the *txt* message is tagged with the number 3 because it ws produced by the instruction on line 3 in Figure 2. Equivalently, one could use variable names (assuming that all variables have distinct names), message

types (assuming that all messages have distinct types), or operators [4]. However, as we will see in the next section, none of these solutions will suffice once we introduce procedures and recursion.

Integrity keys have another advantage over selections: they make it safe for the network to reorder messages. Previous theories and implementations of choreographic languages require a transport protocol that ensures reliable FIFO communication [9, 17]. These models are therefore susceptible to head-of-line blocking [21], where one delayed message can prevent others from being processed (Figure 4a). Figure 4b shows why FIFO is necessary in these models: unordered messages can cause CIVs. Because our model combines unordered messages, integrity keys, and out-of-order processes, it circumvents the head-of-line blocking problem—as shown in Figure 4c.

2.2 Procedural Choreographies

Choreographies can use procedures parameterised on processes for modularity and recursion [7, 17]. Figure 5a shows an example: a procedure X with three *roles* (i.e., process parameters) $\mathsf{a}, \mathsf{b}, \mathsf{c}$. The procedure X is invoked twice—once with processes $\mathsf{p}, \mathsf{q}, \mathsf{r}_1$ (line 7) and again with $\mathsf{p}, \mathsf{q}, \mathsf{r}_2$ (line 8). In the body of X, role a produces a value and sends it to b; then b transforms the value and sends it to c; finally, c processes the value and sends it to a. As usual in most programming languages, we will assume the variables $\mathsf{a}.w, \mathsf{b}.x, \mathsf{c}.y$, and $\mathsf{a}.z$ are locally scoped—this is in contrast to many choreography models [17], where variables at processes are all mutable fields accessible anywhere in the program.

In existing choreography models, a process can only participate in one choreographic procedure at a time. This is no longer the case with fully out-of-order choreographies. Consider Figure 5a, where process p invokes procedure X twice. The process may begin by invoking the first procedure call (line 7), computing $\mathsf{p}.w$ (line 2), and sending $\mathsf{p}.w$ to r_1 (line 3). Then, instead of executing its next instruction—i.e. becoming blocked by waiting for a message on line 6—p can skip the instruction and proceed to invoke the second procedure call (line 8). Thus, we can have an execution like in Figure 5b, in which p sends a message to r_1 as part of the first procedure call and immediately sends a message to r_2 as part of the second procedure call. This unusual semantics is exactly what we would expect in a choreography language with non-blocking receive—such as Choral when using the Ozone API (Section 5).

2.2.1 Interprocedural Integrity

Concurrent choreographic procedures add another dimension of complexity to the communication integrity problem. Figures 5b and 5c show why: depending on the order that r_1 and r_2's messages arrive at q, the messages from q may arrive at p in any order. (This occurs even if we assume reliable FIFO delivery!) Like in the previous section, p cannot distinguish which message pertains to which procedure invocation. But now static information is insufficient to ensure communication integrity: both messages from q pertain to the same variable in the same procedure, so the integrity keys fail to distinguish the different procedure calls. We call this the *interprocedural CIV problem*.

The example above shows that integrity keys need dynamic information prevent CIVs. We can solve the problem by combining the line numbers used in Section 2.1 with some *session token* t that uniquely identifies each procedure invocation. Applied to Figures 5b and 5c, p could inspect the session token to determine whether the messages pertain to the first procedure call (line 7) or the second (line 8). But note that it is insufficient for p and q

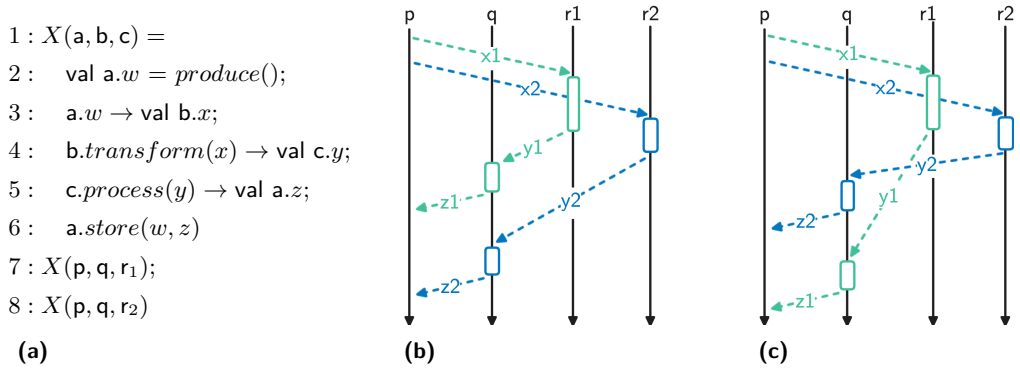


Figure 5 A choreography and two possible executions. In both diagrams, the green lines correspond to $X(\mathsf{p}, \mathsf{q}, \mathsf{r}_1)$ and the blue lines correspond to $X(\mathsf{p}, \mathsf{q}, \mathsf{r}_2)$.

to compute different session tokens for each procedure call; the processes must agree on the same token value for each procedure call.

One solution to the interprocedural CIV problem would be to select a "leader" process for each procedure call, and let the leader compute a session token for all the other roles to use. However, this would make the leader a bottleneck. Instead we propose a method for processes to compute session tokens independently, using only local data, so that the resulting values agree.

Observe that a procedure call is uniquely identified by its caller (i.e. the procedure call that called it) and its line number l. Assuming the caller already has a unique token t, the callee's token can be computed as some injective function $\mathsf{nextToken}(l, t)$. This function would need to satisfy two properties:

- **Determinism:** For any input pair l, t, $\mathsf{nextToken}(l, t)$ always produces the same value t'.
- **Injectivity:** Distinct input pairs l, t produce distinct output tokens.

Determinism ensures that if two processes in the same procedure call (with token t) invoke the same procedure (on line l) then both processes will agree on the value of $\mathsf{nextToken}(l, t)$. *Injectivity* ensures that if a process concurrently participates in two different procedure calls (with distinct tokens t_1, t_2) and invokes two procedures (on lines l_1, l_2—possibly $l_1 = l_2$) then the resulting session tokens will be distinct ($\mathsf{nextToken}(l_1, t_1) \neq \mathsf{nextToken}(l_2, t_2)$). In the next section, we realize these constraints by representing tokens as lists of line numbers and defining nextToken to be the list-prepend operator.

3 Choreography Model

In this section we present O_3, a formal model for asynchronous, fully out-of-order choreographies. Statements can be executed in any order (up to data dependency) and messages can be delivered out of order. The section concludes with proofs of deadlock-freedom and communication integrity.

3.1 Syntax

The syntax for choreographies in O_3 is defined by the grammar in Figure 6. Two example choreographies are shown in Figure 7; we explain their semantics in Section 3.2.2.

$$
\begin{array}{lll|lll}
\mathscr{C} ::= \{X_i(\overline{\mathsf{p}}, \overline{\mathsf{p}.x}) = C_i\}_{i\in\mathcal{I}} & & \textit{(decls)} & & & \\
C ::= I;\, C & & \textit{(seq)} & \mid \{\, C \,\} & & \textit{(block)} \\
\mid 0 & & \textit{(end)} & & & \\
I ::= l, t : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x & & \textit{(comm)} & \mid l, t : \mathsf{p} \to \mathsf{q}[L] & & \textit{(sel)} \\
\mid l, t : \mathsf{val}\ \mathsf{p}.x = e & & \textit{(expr)} & \mid l, t : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2 & & \textit{(cond)} \\
\mid l, t : X(\overline{\mathsf{p}}, \overline{a}) & & \textit{(call)} & \mid l, t : \mathsf{p} \rightsquigarrow \mathsf{q}.x & & \textit{(comm}^\dagger\textit{)} \\
\mid l, t : \mathsf{p} \rightsquigarrow \mathsf{q}[L] & & \textit{(sel}^\dagger\textit{)} & \mid l, t : \overline{\mathsf{p}}.\, X(\overline{\mathsf{q}}, \overline{a})\, \{\, C \,\} & & \textit{(call}^\dagger\textit{)} \\
t ::= \mathsf{t} & & \textit{(placeholder)} & \mid \mathsf{t}_i & & \textit{(token}^\dagger\textit{)} \\
e ::= f(\overline{e}) & & \textit{(app)} & \mid a & & \textit{(atom)} \\
a ::= v@\mathsf{p} & & \textit{(val)} & \mid \mathsf{p}.x & & \textit{(var)} \\
 & & & & & {}^\dagger\textit{runtime only}
\end{array}
$$

Figure 6 Syntax for fully out-of-order choreographies

A choreography C is executed in the context of a collection of *procedures* $\mathscr{C}$. Each procedure $X_i(\overline{\mathsf{p}}, \overline{\mathsf{p}.x}) = C_i$ is parameterized by a list of *roles* $\overline{\mathsf{p}} = \mathsf{p}_1, \ldots, \mathsf{p}_n$ and role-local *parameters* $\overline{\mathsf{p}.x} = \mathsf{p}_{j_1}.x_1, \ldots, \mathsf{p}_{j_m}.x_m$ where every parameter $\mathsf{p}_{j_k}.x_k$ is located at one of the roles in $\overline{\mathsf{p}}$. We assume that procedures do not contain runtime terms (such as $l, t : \mathsf{p} \rightsquigarrow \mathsf{q}.x$).

A choreography C consists of a sequence of *instructions* I, followed by the end symbol 0 which is often omitted. Each instructions is prefixed with a *line number* l and a *token* t. We call this pair an *integrity key*. If C_i is the body of a procedure in $\mathscr{C}$, then the token t on every instruction in C_i must be a *token placeholder* t. When the procedure is invoked, all token placeholders t in C_i will be replaced with a fresh *token value* t_j.

We assume that line numbers in $\mathscr{C}$ are similar to line numbers in a real computer program: Each instruction I in $\mathscr{C}$ has a distinct line number l. When a procedure $X_i(\overline{\mathsf{p}}, \overline{\mathsf{p}.x}) = C_i$ is invoked, the line numbers in C_i will remain unchanged. This will allow us to access the static location of an instruction at runtime in order to compute the integrity key.

Choreographies consist of five kinds of instructions. A communication $\mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x$ instructs process p to evaluate expression e and send it to process q, which will bind the result to $\mathsf{q}.x$. A selection $\mathsf{p} \to \mathsf{q}[L]$ conveys knowledge of choice [17]: it instructs p to send a value literal L to q, informing q that a decision (represented by L) has been made. A local computation $\mathsf{val}\ \mathsf{p}.x = e$ instructs p to evaluate e and bind the result to $\mathsf{p}.x$. A conditional $\mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2$ instructs p to evaluate e and for the processes to proceed with C_1 or C_2 according to the result. A procedure call $X_i(\overline{\mathsf{p}}, \overline{a})$ instructs processes $\overline{\mathsf{p}}$ to invoke procedure $X_i(\overline{\mathsf{q}}, \overline{\mathsf{q}.y}) = C_i$ defined in $\mathscr{C}$, with processes $\overline{\mathsf{p}}$ playing roles $\overline{\mathsf{q}}$ and arguments $\overline{a}$ (which may take the form of values $v@\mathsf{p}$ or variables $\mathsf{p}.x$) substituted for parameters $\overline{\mathsf{q}.y}$. In addition to these basic instructions, a choreography may also contain blocks $\{\, C \,\}$ which limit the scope of variables.

In addition, choreographies can contain *runtime instructions* that represent an instruction in progress; these terms are an artifact of the semantics, not written explicitly by the programmer. A communication-in-progress $\mathsf{p} \rightsquigarrow \mathsf{q}.x$ indicates that p sent a message to q, which q has not yet received. Similarly, a selection-in-progress $\mathsf{p} \rightsquigarrow \mathsf{q}[L]$ indicates that p sent a selection. A procedure-call-in-progress $\overline{\mathsf{p}}.\, X(\overline{\mathsf{q}}, \overline{a})\, \{\, C \,\}$ indicates that some processes have invoked X, and others have not—we leave the details to Section 3.2.

$BuyItem(\mathsf{s},\ \mathsf{b},\ \mathsf{b}.itemID) =$
$1, \mathsf{t} : \mathsf{b}.itemID \to \mathsf{val}\ \mathsf{s}.itemID;$
$2, \mathsf{t} : \mathsf{val}\ \mathsf{s}.item? = sell(\mathsf{s}.itemID);$
$3, \mathsf{t} : \mathsf{s}.item? \to \mathsf{val}\ \mathsf{b}.item?$
$4, \mathsf{t}_0 : BuyItem(\mathsf{seller}, \mathsf{buyer}_1, 123@\mathsf{buyer}_1);$
$5, \mathsf{t}_0 : BuyItem(\mathsf{seller}, \mathsf{buyer}_2, 543@\mathsf{buyer}_2)$

(a)

$StreamIt(\mathsf{p},\ \mathsf{c}) =$
$1, \mathsf{t} : \mathsf{p}.produce() \to \mathsf{val}\ \mathsf{c}.x;$
$2, \mathsf{t} : \mathsf{val}\ \mathsf{c}.z = consume(\mathsf{c}.x);$
$3, \mathsf{t} : \mathsf{if}\ (itemsLeft() > 0)@\mathsf{p}\ \mathsf{then}$
$4, \mathsf{t} : \mathsf{p} \to \mathsf{c}[\textsc{More}]$
$5, \mathsf{t} : StreamIt(\mathsf{p}, \mathsf{c})$
else
$6, \mathsf{t} : \mathsf{p} \to \mathsf{c}[\textsc{Done}]$
$7, \mathsf{t}_0 : StreamIt(\mathsf{p}_1, \mathsf{c});$
$8, \mathsf{t}_0 : StreamIt(\mathsf{p}_2, \mathsf{c})$

(b)

Figure 7 Two example choreographies. On the left, processes buyer_1 and buyer_2 concurrently attempt to buy products from seller. On the right, producers p_1 and p_2 concurrently send streams of data to a shared consumer c.

Expressions e are composed of *atoms* a (i.e. variables $\mathsf{p}.x$ and values $v@p$) and function applications $f(\overline{e})$. We assume that a function f evaluated by p can mutate the state of p as a side-effect. However, the variables $\mathsf{p}.x$ themselves are immutable.

3.2 Semantics

We now give a fully out-of-order semantics for choreographies in O_3. The semantics is a labelled transition system on *configurations* $\langle\, C,\ \Sigma,\ K \,\rangle$, where:

- C is a choreography;
- Σ is a mapping from process names p to process states σ; and
- K is a mapping from process names p to multisets of messages M yet to be delivered to p.

We also assume there exists a set of unchanging procedure declarations $\mathscr{C}$, not shown explicitly in the configuration.

An *initial configuration* is a configuration $\langle\, C,\ \Sigma,\ K \,\rangle$ where Σ maps each p to an arbitrary state, K maps each p to the empty set, and all instructions in C use the same token t_0, called the *initial token*. We also assume the initial configuration is well-formed, cf. Section 3.3. The transition relation $(\xrightarrow{\mathsf{p}})$ is a relation on configurations, where p identifies which process took a step.

Messages in our semantics are represented as triples (l, t_i, v). Here l is the line number of the communication that sent the message, t_i is the token associated with the procedure invocation that sent the message, and v is a value called the *payload*. Together, the pair (l, t_i) is called the *integrity key* of the message.

3.2.1 Transition rules

Figure 8 defines the semantics for O_3, based on textbook models for procedural and asynchronous choreographies. Full out-of-order execution is achieved by weakening the classic C-Delay rule [17]. This yields a semantics in which any pair of statements can be executed

out of order, up to data- and control-dependency. That is, in a choreography of the form $I_1; I_2; C$, the statement I_2 can always be executed before I_1 *unless*:

1. *(Data dependency)* I_1 binds a variable $\mathsf{p}.x$ that is used in I_2; or
2. *(Control dependency)* I_1 is a selection of the form $\mathsf{p} \to \mathsf{q}[L]$ or $\mathsf{p} \rightsquigarrow \mathsf{q}[L]$, and I_2 is an action performed by q.

This semantics overapproximates the concurrency of Choral's Ozone API (Section 5) where actions are only evaluated out of order if they are wrapped in a CompletableFuture.

The semantics for communication is defined by rules C-Send and C-Recv. In C-Send for the communication term $l, \mathsf{t}_i : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x$, the expression e is evaluated in the context of p's state using the notation $\Sigma(\mathsf{p}) \vdash e \Downarrow (v, \sigma)$. Evaluating e produces a value v and a new state σ for p; we assume that $(\vdash)$ is defined for any e that contains no free variables and for any state $\Sigma(\mathsf{p})$. The C-Send rule transforms the communication term into a communication-in-progress term $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ and adds the message (l, t_i, v) to q's set of undelivered messages. The message can subsequently be received by q using the C-Recv rule. This rule removes the communication-in-progress term and substitutes the message payload v into the continuation C. Notice that the integrity key l, t_i of the message is matched against the integrity key of the communication-in-progress, $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$. Notice also that the semantics for communication is not defined if the token t is merely a placeholder t—it must be a token *value* t_i. Indeed, in Section 3.3 we show that placeholders only appear in $\mathscr{C}$, never in C.

Rules C-Select and C-OnSelect closely mirror the semantics of C-Send and C-Recv—the key difference is that a label L is communicated instead of a value. Rules C-Compute and C-If are standard, except for changes made to use lexical scope instead of global scope: C-Compute substitutes the value v into the continuation C (instead of storing it in the local state Σ) and C-If places the continuation C_i in a block to prevent variable capture. To garbage collect empty blocks, C-If uses a concatenation operator $(\fatsemi)$ defined as:

$$\{I; C\} \fatsemi C' = \{I; C\}; C' \qquad\qquad \{0\} \fatsemi C' = C'$$

The C-Delay rule is also standard, except it has been weakened so processes can execute out of order.

C-First, C-Enter, C-Last, and C-Delay-Proc are standard rules for invoking choreographic procedures in a decentralized way [17]. Given a procedure call $l, \mathsf{t}_i : X(\overline{\mathsf{p}}, \overline{a})$, rule C-First indicates that $\mathsf{p} \in \overline{\mathsf{p}}$ has entered the procedure but processes $\overline{\mathsf{p}} \setminus \mathsf{p}$ have yet to enter. The rule replaces the procedure call with a procedure-call-in-progress $l, \mathsf{t}_i : \overline{\mathsf{p}} \setminus \mathsf{p}.\, X(\overline{\mathsf{p}}, \overline{a})\, \{\, C_1' \,\}$ to reflect this fact. The choreography C_1' is the body of the procedure, which p may begin executing via the C-Delay-Proc rule. The remaining processes can enter the procedure via the C-Enter rule, and the last process to enter the procedure uses the C-Last rule.

The key difference between our procedure calls and the standard is our use of nextToken to determine the token used in the procedure. Note that, although nextToken is atomic in the semantics, we show in Section 4 that it can also have a decentralized interpretation. The function $\mathsf{nextToken} : \mathbb{N} \times \mathsf{Token} \to \mathsf{Token}$ is a pure injective function for computing new tokens (of type Token) using integrity keys (of type $\mathbb{N} \times \mathsf{Token}$). To ensure the integrity keys from two concurrent procedures never collide, nextToken must produce unique, non-repeating keys upon iterated application. This is realized by representing $\mathsf{Token} = \mathbb{N}^*$ as lists of numbers, the initial token t_0 as an empty list $[]$, and implementing $\mathsf{nextToken}(l, \mathsf{t}_i) = l :: \mathsf{t}_i$ by prepending the line number l to the list. Intuitively, this means the token associated with a procedure invocation is a simplified *call stack* of line numbers from which the procedure was called.

$$\frac{\Sigma(\mathsf{p}) \vdash e \Downarrow (v, \sigma) \qquad M = K(\mathsf{q}) \uplus \{(l, \mathsf{t}_i, v)\}}{\left\langle\, l, \mathsf{t}_i : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x;\ C,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x;\ C,\ \Sigma[\mathsf{p} \mapsto \sigma],\ K[\mathsf{q} \mapsto M] \,\right\rangle}\ \text{C-Send}$$

$$\frac{(l, \mathsf{t}_i, v) \in K(\mathsf{q}) \qquad M = K(\mathsf{q}) \setminus \{(l, \mathsf{t}_i, v)\}}{\left\langle\, l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x;\ C,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{q}} \left\langle\, C[\mathsf{q}.x \mapsto v@q],\ \Sigma,\ K[\mathsf{q} \mapsto M] \,\right\rangle}\ \text{C-Recv}$$

$$\frac{M = K(\mathsf{q}) \cup \{(l, \mathsf{t}_i, L)\}}{\left\langle\, l, \mathsf{t}_i : \mathsf{p} \to \mathsf{q}[L];\ C,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}[L];\ C,\ \Sigma,\ K[\mathsf{q} \mapsto M] \,\right\rangle}\ \text{C-Select}$$

$$\frac{K(\mathsf{q}) = \{(l, \mathsf{t}_i, L)\} \cup M}{\left\langle\, l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}[L];\ C,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{q}} \left\langle\, C,\ \Sigma,\ K[\mathsf{q} \mapsto M] \,\right\rangle}\ \text{C-OnSelect}$$

$$\frac{\Sigma(\mathsf{p}) \vdash e \Downarrow (v, \sigma)}{\left\langle\, l, \mathsf{t}_i : \mathsf{val}\ \mathsf{p}.x = e;\ C,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, C[\mathsf{p}.x \mapsto v@\mathsf{p}],\ \Sigma[\mathsf{p} \mapsto \sigma],\ K \,\right\rangle}\ \text{C-Compute}$$

$$\frac{\Sigma(\mathsf{p}) \vdash e \Downarrow v \qquad \text{if } v = \mathsf{true} \text{ then } i = 1 \text{ else } i = 2}{\left\langle\, l, \mathsf{t}_i : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2;\ C,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, \{\, C_i \,\} \fatsemi C,\ \Sigma,\ K \,\right\rangle}\ \text{C-If}$$

$$\frac{\left\langle\, C_1,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, C_1',\ \Sigma',\ K' \,\right\rangle}{\left\langle\, \{\, C_1 \,\};\ C_2,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, \{\, C_1' \,\} \fatsemi C_2,\ \Sigma',\ K' \,\right\rangle}\ \text{C-Block}$$

$$\frac{\left\langle\, C,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{q}} \left\langle\, C',\ \Sigma',\ K' \,\right\rangle \qquad I \text{ is not a selection at } \mathsf{q}}{\left\langle\, I;\ C,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{q}} \left\langle\, I;\ C',\ \Sigma',\ K' \,\right\rangle}\ \text{C-Delay}$$

$$\frac{\begin{array}{c}(X(\overline{\mathsf{q}}, \overline{\mathsf{q}.y}) = C_1) \in \mathscr{C} \qquad C_1' = C_1[\overline{\mathsf{q}}, \overline{\mathsf{q}.y}, \mathsf{t} \mapsto \overline{\mathsf{p}}, \overline{a}, \mathsf{t}_j] \\ \mathsf{p} \in \overline{\mathsf{p}} \qquad \mathsf{t}_j = \mathsf{nextToken}(l, \mathsf{t}_i)\end{array}}{\left\langle\, l, \mathsf{t}_i : X(\overline{\mathsf{p}}, \overline{a});\ C_2,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, l, \mathsf{t}_i : \overline{\mathsf{p}} \setminus \mathsf{p}.\, X(\overline{\mathsf{p}}, \overline{a})\, \{\, C_1' \,\};\ C_2,\ \Sigma,\ K \,\right\rangle}\ \text{C-First}$$

$$\frac{\mathsf{p} \in \overline{\mathsf{p}}}{\left\langle\, l, \mathsf{t}_i : \overline{\mathsf{p}}.\, X(\overline{\mathsf{q}}, \overline{a})\, \{\, C_1 \,\};\ C_2,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, l, \mathsf{t}_i : \overline{\mathsf{p}} \setminus \mathsf{p}.\, X(\overline{\mathsf{q}}, \overline{a})\, \{\, C_1 \,\};\ C_2,\ \Sigma,\ K \,\right\rangle}\ \text{C-Enter}$$

$$\frac{}{\left\langle\, l, \mathsf{t}_i : \mathsf{p}.\, X(\overline{\mathsf{q}}, \overline{a})\, \{\, C_1 \,\};\ C_2,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, \{\, C_1 \,\} \fatsemi C_2,\ \Sigma,\ K \,\right\rangle}\ \text{C-Last}$$

$$\frac{\left\langle\, C_1,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, C_1',\ \Sigma',\ K' \,\right\rangle \qquad \mathsf{p} \notin \overline{\mathsf{p}}}{\left\langle\, l, \mathsf{t}_i : \overline{\mathsf{p}}.\, X(\overline{\mathsf{q}}, \overline{a})\, \{\, C_1 \,\};\ C_2,\ \Sigma,\ K \,\right\rangle \xrightarrow{\mathsf{p}} \left\langle\, l, \mathsf{t}_i : \overline{\mathsf{p}}.\, X(\overline{\mathsf{q}}, \overline{a})\, \{\, C_1' \,\};\ C_2,\ \Sigma',\ K' \,\right\rangle}\ \text{C-Delay-Proc}$$

Figure 8 Semantics for fully out-of-order choreographies

3.2.2 Discussion

Figure 7a expresses a choreography in which two buyer processes concurrently buy items from a seller process. In the initial configuration, buyer_1 can enter the procedure on line 4, buyer_2 can enter the procedure on line 5, and seller can enter either procedure. If buyer_2 enters first (using C-Delay and C-Enter), it can proceed to send $543@\mathsf{buyer}_2$ to seller (using C-Com). Then seller can enter the procedure on line 5 (using C-Delay and C-Last) and proceed to receive the message from buyer_2 (using C-Recv). This execution would be impossible in a standard choreography model because seller would need to complete the procedure invocation on line 4 before it could enter the procedure on line 5. The added concurrency ensures that slowness in buyer_1 does not prevent buyer_2 from making progress.

Note that the out-of-order semantics of Figure 7a also adds nondeterminism. Suppose buyer_1 and buyer_2 attempt to buy the same item and the seller only has one copy in stock. One of the buyers will receive the item, and the other buyer will receive a null value. In a standard choreography model, the item would always go to buyer_1. In O_3, the item will be sold nondeterministically according to the order that messages arrive to the seller. This nondeterminism can be problematic—it makes reasoning about choreographies harder—but also increases expressivity: nondeterminism is essential in distributed algorithms like consensus and leader election. Reasoning about nondeterminism in choreographies is an important topic for future work.

Figure 7b shows we can also express recursive choreographies. In each iteration of the procedure *StreamIt*, a producer p sends a value to a consumer c (line 1) and decides whether to start another iteration (line 3). Then the producer asynchronously informs the consumer about its decision (lines 4 and 6) and can proceed with the next iteration (line 5) without waiting for the consumer. Because messages in O_3 are unordered, the consumer can consume items (line 2) from different iterations in any order; this prevents head-of-line blocking [21].

In the initial choreography of Figure 7b, producers $\mathsf{p}_1, \mathsf{p}_2$ and a consumer c invoke two instances of *StreamIt*. As in Figure 7a, the two procedures evolve concurrently; a slowdown in p_1 will not prevent c from consuming items produced by p_2.

Conspicuously absent from our semantics is the standard C-Delay-Cond rule [17], which might be written as follows:

$$\frac{\langle C_1,\ \Sigma,\ K\rangle \xrightarrow{\mathsf{p}} \langle C_1',\ \Sigma',\ K'\rangle \qquad \langle C_2,\ \Sigma,\ K\rangle \xrightarrow{\mathsf{p}} \langle C_2',\ \Sigma',\ K'\rangle}{\langle l, \mathsf{t}_i : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2;\ C,\ \Sigma,\ K\rangle \xrightarrow{\mathsf{p}} \langle l, \mathsf{t}_i : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1'\ \mathsf{else}\ C_2';\ C,\ \Sigma',\ K'\rangle}$$

This rule allows one to evaluate the branches of an if-instruction before the guard e. In O_3, it is not clear how to add such a rule without violating the EPP Theorem (Theorem 6): the rule would allow the following reduction (omitting Σ and K for simplicity):

$$\begin{pmatrix} 1, \mathsf{t}_0 : \mathsf{if}\ e@\mathsf{q}\ \mathsf{then} \\ \quad 2, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.x = \mathit{compute}(); \\ \quad 3, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.y = \mathit{compute}(); \\ \quad 4, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.z = \mathit{print}(\mathsf{p}.x) \\ \mathsf{else} \\ \quad 5, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.x = \mathit{compute}(); \\ \quad 6, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.y = \mathit{compute}(); \\ \quad 7, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.z = \mathit{print}(\mathsf{p}.x) \end{pmatrix} \xrightarrow{\mathsf{p}} \begin{pmatrix} 1, \mathsf{t}_0 : \mathsf{if}\ e@\mathsf{q}\ \mathsf{then} \\ \quad 3, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.y = \mathit{compute}(); \\ \quad 4, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.z = \mathit{print}(v) \\ \mathsf{else} \\ \quad 5, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.x = \mathit{compute}(); \\ \quad 7, \mathsf{t}_0 : \mathsf{val}\ \mathsf{p}.z = \mathit{print}(\mathsf{p}.x) \end{pmatrix}$$

The resulting choreography allows $\mathit{print}(v)$ to be evaluated in the "then" branch, but not in the "else" branch. Thus the resulting choreography is not projectable.

Instead of attempting to accommodate the C-Delay-Cond rule, we follow prior work [10, 22, 19] and remove the rule entirely. This decision slightly simplifies the definition of endpoint projection (Section 4.3) at the cost of additional communication in certain cases.

3.3 Properties

In this section we prove that O_3 choreographies are deadlock-free and we formalize the communication integrity property. Combined with the EPP Theorem presented in Section 4, these results imply that projected code inherits the same properties.

We restrict our attention to configurations that are reachable from the initial configuration. For example, the following configurations are not reachable:

$$\langle\, l, \mathsf{t}_0 : \mathsf{p} \rightsquigarrow \mathsf{q}.x,\ \Sigma,\ \{\mathsf{p} \mapsto \emptyset, \mathsf{q} \mapsto \emptyset\} \,\rangle$$

$$\langle\, l, \mathsf{t}_0 : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x,\ \Sigma,\ \{\mathsf{p} \mapsto \emptyset, \mathsf{q} \mapsto \{(l, \mathsf{t}_i, v)\}\} \,\rangle$$

$$\langle\, \{1, \mathsf{t}_1 : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x\};\ \{1, \mathsf{t}_1 : \mathsf{p}.e' \to \mathsf{val}\ \mathsf{q}.x\},\ \Sigma,\ \{\mathsf{p} \mapsto \emptyset, \mathsf{q} \mapsto \emptyset\} \,\rangle$$

$$\langle\, 3, \mathsf{t}_0 : \mathsf{p}.\, X(\mathsf{p}, \mathsf{q})\, \{\, 1, \mathsf{t}_0 : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x \,\},\ \Sigma,\ \{\mathsf{p} \mapsto \emptyset, \mathsf{q} \mapsto \emptyset\} \,\rangle$$

The first configuration is not reachable because $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ never occurs unless q has an undelivered message from p. Dually, the second configuration is not reachable because p has a message in its queue that, according to the choreography, has not yet been sent. The third configuration is unreachable because the two instructions share the same integrity key; we will show that $\mathsf{nextToken}$ ensures such configurations never arise. Likewise, $\mathsf{nextToken}$ also forbids the last configuration, since the token of the instruction $1, \mathsf{t}_0 : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x$ must have been derived from the integrity key of the enclosing call $3, \mathsf{t}_0 : \mathsf{p}.\, X(\mathsf{p}, \mathsf{q})\, \{\ldots\}$. Specifically, $\mathsf{t}_0 \neq \mathsf{nextToken}(3, \mathsf{t}_0)$.

To specify the last property above, recall that tokens are represented as lists of integers $l_1 :: l_2 :: \ldots$. We say (l_1, t_1) is a *prefix* of (l_2, t_2)—written $(l_1, t_1) \prec (l_2, t_2)$—if the list $l_1 :: t_1$ is a prefix of $l_2 :: t_2$ and we say the keys are *disjoint* if neither is a prefix of the other.

We formalize the properties of reachable configurations by defining which configurations are *well-formed*. Our notion of well-formedness expands on the standard definition [17] in the following ways:

1. A communication-in-progress $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ occurs in C if and only if $(l, \mathsf{t}_i, v) \in K(\mathsf{q})$ for some v.
2. A selection-in-progress $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}[L]$ occurs in C if and only if $(l, \mathsf{t}_i, L) \in K(\mathsf{q})$.
3. Each instruction I in C has a distinct integrity key l, t, where t is a token value (not a placeholder).
4. If the integrity key of I is a prefix of the integrity key of I' then I is a communication-in-progress $l, t : \overline{\mathsf{p}}.\, X(\overline{\mathsf{p}}, \overline{a})\, \{\, C' \,\}$ and I' is in C.

A full definition of well-formedness is presented in Appendix A.

▶ **Theorem 1** (Preservation). *If* $\langle\, C,\ \Sigma,\ K \,\rangle$ *is well-formed and* $\langle\, C,\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, C',\ \Sigma',\ K' \,\rangle$, *then* $\langle\, C',\ \Sigma',\ K' \,\rangle$ *is well-formed.*

Proof. By induction on the definition of $\xrightarrow{\mathsf{p}}$. We focus on the rules for communication and procedure invocation.

C-Send replaces a term $l, \mathsf{t}_i : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x$ with $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ and adds a message (l, t_i, v). By the induction hypothesis, (l, t_i, v) is not already in K.

C-Recv eliminates the runtime term $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ and removes a message (l, t_i, v). By the induction hypothesis, no other $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ term occurs in C.

C-First introduces new terms into the choreography by invoking the call $l_1, \mathsf{t}_1 : X(\overline{\mathsf{p}}, \overline{a})$. By the induction hypothesis, for any other instruction $l_2, t_2 : I$ in C, either (a) keys l_1, t_1 and l_2, t_2 are disjoint; or (b) $l_2, t_2 : I$ is a call-in-progress containing $l_1, \mathsf{t}_1 : X(\overline{\mathsf{p}}, \overline{a})$. In case (a), disjointness implies any instruction in the body of the procedure $C'[\overline{\mathsf{q}}, \overline{\mathsf{q}.y}, \mathsf{t} \mapsto \overline{\mathsf{p}}, \overline{\mathsf{p}.x}, \mathsf{t}_j]$ will also have a key that is disjoint from l_2, t_2. In case (b), notice $\forall l$, $(l_2, t_2) \prec (l_1, t_1) \prec (l, \mathsf{nextToken}(l_1, t_1))$; hence any interaction in the body has a key of which (l_2, t_2) is a prefix. ◀

▶ **Theorem 2** (Deadlock-Freedom). *If* $\langle C,\ \Sigma,\ K \rangle$ *is well-formed, then either* $C \equiv 0$ *or* $\langle C,\ \Sigma,\ K \rangle \xrightarrow{\mathsf{p}} \langle C',\ \Sigma',\ K' \rangle$ *for some* $\mathsf{p}, C', \Sigma', K'$.

Proof. By induction on the structure of C, making use of the full definition of well-formedness in Appendix A. In each case, we observe the first instruction I of C can always be executed. For instance, if $I \equiv l, \mathsf{t}_i : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x$ then the C-Send rule can be applied because well-formedness implies e has no free variables. If $I \equiv l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$, there must be a message $(l, \mathsf{t}_i, v) \in K(\mathsf{q})$ because the configuration is well-formed. The other cases follow similarly. ◀

We end this section with a formalization of communication integrity. Consider the buggy execution in Figure 2: in a model without integrity keys, the execution reaches a configuration

$$\langle \mathsf{s} \rightsquigarrow \mathsf{c}.txt; \mathsf{s} \rightsquigarrow \mathsf{c}.key; \ldots,\ \Sigma,\ \mathsf{c} \mapsto v_{key}, v_{txt} \rangle,$$

where v_{key} is the value produced by $\mathsf{ks}.getKey()$ and v_{txt} is the value produced by $\mathsf{cs}.getText()$. A CIV occurs if the configuration can make a transition that consumes $\mathsf{s} \rightsquigarrow \mathsf{c}.txt$ and v_{key} together, binding $\mathsf{c}.txt$ to v_{key}. We therefore want to ensure:

- There is only one way a communication-in-progress instruction can be consumed; and
- The instruction is consumed together with the correct message.

▶ **Definition 3** (Send/receive transitions). *A* send transition $\langle C,\ \Sigma,\ K \rangle \xrightarrow{\mathsf{p}} \langle C',\ \Sigma',\ K' \rangle$ *is a transition with a derivation that ends with an application of* C-Send. *Likewise, a* receive transition *is a transition with a derivation that ends with* C-Recv.

▶ **Theorem 4** (Communication Integrity). *Let* $e = c_0 \xrightarrow{\mathsf{p}_1} \cdots \xrightarrow{\mathsf{p}_{k+1}} c_{k+1}$ *be an execution ending with a send transition* $c_k \xrightarrow{\mathsf{p}} c_{k+1}$, *which produces instruction* $l, \mathsf{t}_k : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ *and message* m. *Let* $e' = c_0 \xrightarrow{\mathsf{p}_1} \cdots \xrightarrow{\mathsf{p}_n} c_n$ $(n > k)$ *be an execution extending* e, *where* $l, \mathsf{t}_k : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ *has not yet been consumed. Then there is at most one receive transition* $c_n \xrightarrow{\mathsf{q}} c_{n+1}$ *consuming* $l, \mathsf{t}_k : \mathsf{p} \rightsquigarrow \mathsf{q}.x$. *Namely, it is the transition that consumes* $l, \mathsf{t}_k : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ *and* m *together.*

Proof. By definition of C-Send, m has the form (l, t_k, v). By definition of C-Recv, if there exists a transition $c_n \to c_{n+1}$ that consumes $l, \mathsf{t}_k : \mathsf{p} \rightsquigarrow \mathsf{q}.x$, then the transition also consumes a message (l, t_k, v'), for some v'. It therefore suffices to show the message (l, t_k, v') is unique and that $v' = v$. This follows by induction on the length m of the extension:

- *Base case:* Well-formedness implies there is no message (l, t_k, v') in c_k. Hence the message (l, t_k, v) in c_{k+1} is unique.
- *Induction step:* Observe that the transition $c_m \to c_{m+1}$ cannot remove (l, t_k, v); this would require consuming $l, \mathsf{t}_k : \mathsf{p} \rightsquigarrow \mathsf{q}.x$, which cannot happen in e' by hypothesis. Also observe that the transition cannot add a new message with integrity key (l, t_k); this would require consuming an instruction $l, \mathsf{t}_k : \mathsf{p}'.e \to \mathsf{val}\ \mathsf{q}.x'$, which cannot exist in c_m by well-formedness. Hence (l, t_k, v) is unique in c_{m+1}.

◀

$$
\begin{array}{rlll}
\mathscr{P} & ::= \{X_i(\overline{\mathsf{p}}_i, \overline{x}_i) = C_i\}_{i\in\mathcal{I}} & \textit{(decls)} & \\
P, Q & ::= I;\ P & \textit{(seq)} & \mid \{\, P \,\} \quad \textit{(block)} \\
& \mid 0 & \textit{(end)} & \\
I & ::= \mathsf{p}\,!_{l,t}\, e & \textit{(send)} & \mid ?_{l,t}\, x \quad \textit{(receive)} \\
& \mid \mathsf{val}\ x = e & \textit{(expr)} & \mid \mathsf{p} \oplus_{l,t} L \quad \textit{(choice)} \\
& \mid \&\{(l_i, \mathsf{t}_i, L_i) \Rightarrow P_i\}_{i\in\mathcal{I}} & \textit{(branch)} & \mid \mathsf{if}\ e\ \mathsf{then}\ P\ \mathsf{else}\ Q \quad \textit{(cond)} \\
& \mid l, t : X(\overline{\mathsf{p}}, \overline{a}) & \textit{(call)} & \\
e & ::= f(\overline{e}) & \textit{(app)} & \mid a \quad \textit{(atom)} \\
a & ::= x & \textit{(var)} & \mid v \quad \textit{(val)} \\
N, M & ::= \mathsf{p}[P] & \textit{(proc)} & \mid (N \mid M) \quad \textit{(par)}
\end{array}
$$

Figure 9 Syntax for out-of-order processes

4 Process Model

4.1 Syntax

Figure 9 presents the syntax for out-of-order processes. A term $\mathsf{p}[P]$ is a process named p with behavior P. Networks, ranged over by N, M, are parallel compositions of processes. Compared to prior work [17], certain process instructions need to be annotated with integrity keys (for instance, message send $\mathsf{p}\,!_{l,t}\, e$ and procedure call $l, t : X(\overline{\mathsf{p}}, \overline{a})$). In addition, when receiving a message it is no longer necessary to specify a sender—for instance, it suffices to write $?_{l,t}\, x;\ P$ instead of the more traditional $\mathsf{p}\,?_{l,t}\, x;\ P$.

4.2 Semantics

The semantics for out-of-order processes is presented in Figure 10. It is defined as a labelled transition system on *process configurations* $\langle\, N,\ \Sigma,\ K \,\rangle$, where N is a network and Σ, K have the same meaning as in Section 3.2. We also let $\mathscr{P}$ be an implicit set of procedure declarations.

The transition rules of Figure 10 are similar to prior work. P-Send adds a message (l, t_i, v) to the undelivered messages of q, whereas P-Recv removes the message and substitutes it into the body of the process. Similarly, P-Select adds (l, t_i, L) to the message set and P-OnSelect selects a branch from the set of options $\&\{(l_j, \mathsf{t}_j, L_j) \Rightarrow P_j\}_{j\in\mathcal{J}}$. P-Call invokes a procedure, locally computing the next token and substituting the body of the procedure into the process. Rules P-Compute, P-If, and P-Par are standard.

The key novelty of out-of-order processes is the P-Delay rule, which allows a process to perform instructions in *any* order, up to data- and control-dependencies. The latter implies processes cannot evaluate instructions nested within an if or $\&$-expression.

4.3 Endpoint Projection

Figure 12 defines the *endpoint projection (EPP)* $[\![C]\!]$ of a choreography C, translating it into a network. All the rules follow from trivial modifications of the standard EPP for procedural choreographies [17], except the definition of *merging* ($\sqcup$). Merging is used to define the EPP for conditionals $[\![\mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2]\!]_{\mathsf{r}}$ when $\mathsf{r} \neq \mathsf{p}$; the process $[\![C_1]\!]_{\mathsf{r}} \sqcup [\![C_2]\!]_{\mathsf{r}}$ is one that may behave like $[\![C_1]\!]_{\mathsf{r}}$ or like $[\![C_2]\!]_{\mathsf{r}}$, depending on p's decision. In our model, merging must only be defined when $[\![C_1]\!]_{\mathsf{r}}$ and $[\![C_2]\!]_{\mathsf{r}}$ are both branching processes of the

$$\frac{\Sigma(\mathsf{p}) \vdash e \Downarrow (v, \sigma) \qquad M = K(\mathsf{q}) \uplus \{(l, \mathsf{t}_i, v)\}}{\langle\, \mathsf{p}[\mathsf{q}\,!_{l,\mathsf{t}_i}\, e;\, P],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[P],\ \Sigma[\mathsf{p} \mapsto \sigma],\ K[\mathsf{q} \mapsto M] \,\rangle}\ \text{P-Send}$$

$$\frac{(l, \mathsf{t}_i, v) \in K(\mathsf{q}) \qquad M = K(\mathsf{q}) \setminus \{(l, \mathsf{t}_i, v)\}}{\langle\, \mathsf{q}[?_{l,\mathsf{t}_i}\, x;\, Q],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{q}} \langle\, \mathsf{q}[Q[x \mapsto v]],\ \Sigma,\ K[\mathsf{q} \mapsto M] \,\rangle}\ \text{P-Recv}$$

$$\frac{M = K(\mathsf{q}) \cup \{(l, \mathsf{t}_i, L)\}}{\langle\, \mathsf{p}[\mathsf{q} \oplus_{l,\mathsf{t}_i} L;\, P],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[P],\ \Sigma,\ K[\mathsf{q} \mapsto M] \,\rangle}\ \text{P-Select}$$

$$\frac{K(\mathsf{q}) = \{(l_i, \mathsf{t}_i, L_i)\} \cup M \qquad i \in \mathcal{I}}{\langle\, \mathsf{q}[\&\{(l_j, \mathsf{t}_j, L_j) \Rightarrow Q_j\}_{j \in \mathcal{I}};\, Q],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{q}} \langle\, \mathsf{q}[\{Q_i\};\, Q],\ \Sigma,\ K[\mathsf{q} \mapsto M] \,\rangle}\ \text{P-OnSelect}$$

$$\frac{\Sigma(\mathsf{p}) \vdash e \Downarrow (v, \sigma)}{\langle\, \mathsf{p}[\mathsf{val}\ x = e;\, P],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[P[x \mapsto v]],\ \Sigma[\mathsf{p} \mapsto \sigma],\ K \,\rangle}\ \text{P-Compute}$$

$$\frac{\Sigma(\mathsf{p}) \vdash e \Downarrow v \qquad \text{if } v = \mathsf{true} \text{ then } i = 1 \text{ else } i = 2}{\langle\, \mathsf{p}[\mathsf{if}\ e\ \mathsf{then}\ P_1\ \mathsf{else}\ P_2;\, P],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[\{P_i\};\, P],\ \Sigma,\ K \,\rangle}\ \text{P-If}$$

$$\frac{\langle\, \mathsf{p}[P_1],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[P_1'],\ \Sigma',\ K' \,\rangle}{\langle\, \mathsf{p}[\{P_1\};\, P_2],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[\{P_1'\};\, P_2],\ \Sigma',\ K' \,\rangle}\ \text{P-Block}$$

$$\frac{\langle\, \mathsf{p}[P],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[P'],\ \Sigma',\ K' \,\rangle}{\langle\, \mathsf{p}[I;\, P],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[I;\, P'],\ \Sigma',\ K' \,\rangle}\ \text{P-Delay}$$

$$\frac{(X(\overline{\mathsf{q}}, \overline{y}) = Q) \in \mathscr{P} \qquad \mathsf{nextToken}(l, \mathsf{t}_i) = \mathsf{t}_j}{\langle\, \mathsf{p}[l, \mathsf{t}_i : X(\overline{\mathsf{p}}, \overline{a});\, P],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, \mathsf{p}[\{Q[\overline{\mathsf{q}}, \overline{y}, \mathsf{t} \mapsto \overline{\mathsf{p}}, \overline{a}, \mathsf{t}_j]\};\, P],\ \Sigma,\ K \,\rangle}\ \text{P-Call}$$

$$\frac{\langle\, N,\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, N',\ \Sigma',\ K' \,\rangle}{\langle\, N \mid M,\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, N' \mid M,\ \Sigma',\ K' \,\rangle}\ \text{P-Par}$$

Figure 10 Semantics of out-of-order processes

$$
\begin{array}{l}
\mathit{BuyItem}_1(\mathsf{b}) = \\
\quad ?_{1,\mathsf{t}}\, \mathit{itemID}; \\
\quad \mathsf{val}\ \mathit{item?} = \mathit{sell}(\mathit{itemID}); \\
\quad \mathsf{b}\,!_{3,\mathsf{t}}\, \mathit{item?} \\
\mathit{BuyItem}_2(\mathsf{s},\ \mathit{itemID}) = \\
\quad \mathsf{s}\,!_{1,\mathsf{t}}\, \mathit{itemID}; \\
\quad ?_{3,\mathsf{t}}\, \mathit{item?} \\
\mathsf{seller}[4, \mathsf{t}_0 : \mathit{BuyItem}_1(\mathsf{buyer}_1); \\
\quad 5, \mathsf{t}_0 : \mathit{BuyItem}_1(\mathsf{buyer}_2)] \mid \\
\mathsf{buyer}_1[4, \mathsf{t}_0 : \mathit{BuyItem}_2(\mathsf{seller}, 123)] \mid \\
\mathsf{buyer}_2[5, \mathsf{t}_0 : \mathit{BuyItem}_2(\mathsf{seller}, 543)]
\end{array}
$$

(a)

$$
\begin{array}{l}
\mathit{StreamIt}_1(\mathsf{c}) = \\
\quad \mathsf{c}\,!_{1,\mathsf{t}}\, \mathit{produce}(); \\
\quad \mathsf{if}\ (\mathit{itemsLeft}() > 0)\ \mathsf{then} \\
\quad\quad \mathsf{c} \oplus_{4,\mathsf{t}} \textsc{More};\ 5, \mathsf{t} : \mathit{StreamIt}_1(\mathsf{c}) \\
\quad \mathsf{else}\ \ \mathsf{c} \oplus_{6,\mathsf{t}} \textsc{Done} \\
\mathit{StreamIt}_2(\mathsf{p}) = \\
\quad ?_{1,\mathsf{t}}\, x;\ \mathsf{val}\ z = \mathit{consume}(x); \\
\quad \&\,\{(4, \mathsf{t}, \textsc{More}) \Rightarrow 5, \mathsf{t} : \mathit{StreamIt}_2(\mathsf{p}), \\
\quad\quad (6, \mathsf{t}, \textsc{Done}) \Rightarrow 0\} \\
\mathsf{p}_1[7, \mathsf{t}_0 : \mathit{StreamIt}_1(\mathsf{c})] \mid \\
\mathsf{p}_2[8, \mathsf{t}_0 : \mathit{StreamIt}_1(\mathsf{c})] \mid \\
\mathsf{c}[7, \mathsf{t}_0 : \mathit{StreamIt}_2(\mathsf{p});\ 8, \mathsf{t}_0 : \mathit{StreamIt}_2(\mathsf{p})]
\end{array}
$$

(b)

Figure 11 Projected processes from Figure 7.

form $\&\{(l_i, \mathsf{t}_i, L_i) \Rightarrow P_i\}_{i \in \mathcal{I}}$. In models with C-Delay-Cond (c.f. Section 3.2.2) merging must accommodate other kinds of processes.

Figure 11 shows networks projected from the choreographies of Figure 7. Notice the choreographic procedures *BuyItem* and *StreamIt* are each split into two process procedures—one for each role. Communications in the choreography are, as usual, projected into send and receive instructions. Conditionals in the choreography are projected into an if-instruction at one process and a branch-instruction at the other processes awaiting its decision.

Below we formulate the hallmark *EPP Theorem*, which states that a choreography C and its projection $[\![C]\!]$ evolve in lock-step, up to the usual ($\sqsupseteq$) relation [17]. The proof technique is standard, although our result requires checking many more cases because of the extra concurrency inherent in the model. The statement of the theorem itself is slightly modified, restricting attention to only the *well-formed* networks. We say that a network N is well-formed if the keys in each process are distinct, i.e. $\mathsf{keys}(P)$ is distinct for each $\mathsf{p}[P]$ in N, where $\mathsf{keys}(P)$ is defined in Figure 13. The restriction allows us to consider only networks reachable from the initial configuration, ignoring ill-formed processes such as

$$\mathsf{p}[\&\{(1, \mathsf{t}_0, L) \Rightarrow P_1\};\ \&\{(1, \mathsf{t}_0, L) \Rightarrow P_2\}].$$

▶ **Lemma 5.**

1. $[\![C[\mathsf{p}.x \mapsto v]]\!]_{\mathsf{p}} = [\![C]\!]_{\mathsf{p}}[x \mapsto v]$.
2. $[\![C[\mathsf{p}.x \mapsto v]]\!]_{\mathsf{q}} = [\![C]\!]_{\mathsf{q}}$ *if* $p \neq q$.
3. *If* $P \sqsupseteq Q$ *then* $P[x \mapsto v] \sqsupseteq Q[x \mapsto v]$.
4. $[\![I;\, C]\!]_{\mathsf{q}} = [\![I]\!]_{\mathsf{q}} \fatsemi [\![C]\!]_{\mathsf{q}}$, *where* $(\fatsemi)$ *is the concatenation operator on choreographies, if* I *is not a selection at* q. *That is,* I *does not have the form* $l, t : \mathsf{p} \to \mathsf{q}[L]$ *or* $l, t : \mathsf{p} \rightsquigarrow \mathsf{q}[L]$.
5. *If* $P \sqsupseteq [\![C]\!]_{\mathsf{p}}$ *then* $\mathsf{keys}(P) \supseteq \mathsf{keys}_{\mathsf{p}}(C)$.

▶ **Theorem 6** (EPP Theorem). *Let* $\langle\, C,\ \Sigma,\ K \,\rangle$ *be a well-formed configuration.*

1. *(Completeness) If* $\langle\, C,\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, C',\ \Sigma',\ K' \,\rangle$ *then* $\langle\, [\![C]\!],\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, N',\ \Sigma',\ K' \,\rangle$ *for some well-formed* N' *where* $N' \sqsupseteq [\![C']\!]$.

$$[\![\mathscr{C}]\!] = \bigcup_{i\in\mathcal{I}} [\![X_i(\overline{\mathsf{p}}, \overline{\mathsf{p}.x}) = C_i]\!]$$

$$[\![X_i(\overline{\mathsf{p}}, \overline{\mathsf{p}.x}) = C_i]\!] = \{X_{i,j}(\overline{\mathsf{p}} \setminus \mathsf{p}_j, [\![\overline{\mathsf{p}.x}]\!]_{p_j}) = [\![C_i]\!]_{\mathsf{p}_j} \mid \overline{\mathsf{p}} = \mathsf{p}_1, \ldots, \mathsf{p}_n,\ j \le n\}$$

$$[\![l, t : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x;\ C]\!]_\mathsf{r} = \begin{cases} \mathsf{q}\,!_{l,t}\,e;\ [\![C]\!]_\mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ ?_{l,t}\,x;\ [\![C]\!]_\mathsf{r} & \text{if } \mathsf{r} = \mathsf{q} \\ [\![C]\!]_\mathsf{r} & \text{otherwise} \end{cases}$$

$$[\![l, t : \mathsf{p} \rightsquigarrow \mathsf{q}.x;\ C]\!]_\mathsf{r} = \begin{cases} ?_{l,t}\,x;\ [\![C]\!]_\mathsf{r} & \text{if } \mathsf{r} = \mathsf{q} \\ [\![C]\!]_\mathsf{r} & \text{otherwise} \end{cases}$$

$$[\![l, t : \mathsf{val}\ \mathsf{p}.x = e;\ C]\!]_\mathsf{r} = \begin{cases} \mathsf{val}\ x = [\![e]\!]_\mathsf{r};\ [\![C]\!]_\mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ [\![C]\!]_\mathsf{r} & \text{otherwise} \end{cases}$$

$$[\![l, t : \mathsf{p} \to \mathsf{q}[L];\ C]\!]_\mathsf{r} = \begin{cases} \mathsf{q} \oplus_{l,t} [\![e]\!]_\mathsf{r};\ [\![C]\!]_\mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ \&\{(l, t, L) \Rightarrow [\![C]\!]_\mathsf{r}\} & \text{if } \mathsf{r} = \mathsf{q} \\ [\![C]\!]_\mathsf{r} & \text{otherwise} \end{cases}$$

$$[\![l, t : \mathsf{p} \rightsquigarrow \mathsf{q}[L];\ C]\!]_\mathsf{r} = \begin{cases} \&\{(l, t, L) \Rightarrow [\![C]\!]_\mathsf{r}\} & \text{if } \mathsf{r} = \mathsf{q} \\ [\![C]\!]_\mathsf{r} & \text{otherwise} \end{cases}$$

$$[\![l, t : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2;\ C]\!]_\mathsf{r} = \begin{cases} \mathsf{if}\ [\![e]\!]_\mathsf{r}\ \mathsf{then}\ [\![C_1]\!]_\mathsf{r}\ \mathsf{else}\ [\![C_2]\!]_\mathsf{r};\ [\![C]\!]_\mathsf{r} & \text{if } \mathsf{r} = \mathsf{p} \\ [\![C_1]\!]_\mathsf{r} \sqcup [\![C_2]\!]_\mathsf{r};\ [\![C]\!]_\mathsf{r} & \text{if } \mathsf{r} \in \mathsf{pn}(C_1, C_2) \setminus \mathsf{p} \\ [\![C]\!]_\mathsf{r} & \text{otherwise} \end{cases}$$

$$[\![l, t : X_i(\overline{\mathsf{p}}, \overline{a});\ C]\!]_\mathsf{r} = \begin{cases} l, t : X_{i,j}(\overline{\mathsf{p}} \setminus \mathsf{p}_j, [\![\overline{a}]\!]_{\mathsf{p}_j});\ [\![C]\!]_{\mathsf{p}_j} & \text{if } \mathsf{r} = \mathsf{p}_j \text{ where } \overline{\mathsf{p}} = \mathsf{p}_1, \ldots, \mathsf{p}_n \\ [\![C]\!]_\mathsf{r} & \text{otherwise} \end{cases}$$

$$[\![l, t : \overline{\mathsf{q}}.\ X_i(\overline{\mathsf{p}}, \overline{a})\,\{\,C_1\,\};\ C_2]\!]_\mathsf{r} = \begin{cases} l, t : X_{i,j}(\overline{\mathsf{p}} \setminus \mathsf{p}_j, [\![\overline{a}]\!]_{\mathsf{p}_j});\ [\![C_2]\!]_{\mathsf{p}_j} & \text{if } \mathsf{r} \in \overline{\mathsf{q}} \text{ and } \mathsf{r} = \mathsf{p}_j \\ [\![C_1;\ C_2]\!]_r & \text{if } \mathsf{r} \in \overline{\mathsf{p}} \setminus \overline{\mathsf{q}} \\ [\![C_2]\!]_\mathsf{r} & \text{otherwise} \end{cases}$$

$$[\![\{\,C_1\,\};\ C_2]\!]_\mathsf{r} = \{[\![C_1]\!]_\mathsf{r}\};[\![C_2]\!]_\mathsf{r} \qquad [\![v@\mathsf{p}]\!]_\mathsf{r} = \begin{cases} v & \text{if } \mathsf{r} = \mathsf{p} \\ \bot & \text{otherwise} \end{cases}$$

$$[\![a_1, \ldots, a_n]\!]_\mathsf{r} = [\![a_1]\!]_\mathsf{r}, \ldots, [\![a_n]\!]_\mathsf{r}$$

$$[\![f(e_1, \ldots, e_n)]\!]_\mathsf{r} = f([\![e_1]\!]_\mathsf{r}, \ldots, [\![e_n]\!]_\mathsf{r}) \qquad [\![\mathsf{p}.x]\!]_\mathsf{r} = \begin{cases} x & \text{if } \mathsf{r} = \mathsf{p} \\ \bot & \text{otherwise} \end{cases}$$

$$(\&\{(l_i, \mathsf{t}_i, L_i) \Rightarrow P_i\}_{i\in\mathcal{I}}) \sqcup (\&\{(l_j, \mathsf{t}_j, L_j) \Rightarrow P_j\}_{j\in\mathcal{J}}) = \&\{(l_k, \mathsf{t}_k, L_k) \Rightarrow P_k\}_{k\in\mathcal{I}\cup\mathcal{J}} \quad \text{if } \{L_i : i \in \mathcal{I}\} \# \{L_j : j \in \mathcal{J}\}$$

$$0 \sqsupseteq 0$$

$$(P_1;\ P_2) \sqsupseteq (Q_1;\ Q_2) \quad \text{if } P_i \sqsupseteq Q_i \text{ for } i = 1, 2$$

$$(\mathsf{if}\ e\ \mathsf{then}\ P_1\ \mathsf{else}\ P_2) \sqsupseteq (\mathsf{if}\ e\ \mathsf{then}\ Q_1\ \mathsf{else}\ Q_2) \quad \text{if } P_i \sqsupseteq Q_i \text{ for } i = 1, 2$$

$$I_1 \sqsupseteq I_2 \quad \text{if } I_1 = I_2 \text{ or } I_1 = I_1 \sqcup I_2$$

Figure 12 Endpoint projection

$$\mathsf{keys}(0) = \epsilon$$
$$\mathsf{keys}(I;\ P) = \mathsf{keys}(I), \mathsf{keys}(P)$$
$$\mathsf{keys}(\mathsf{p}\,!_{l,t}\,e) = (l,t)$$
$$\mathsf{keys}(?_{l,t}\,x) = (l,t)$$
$$\mathsf{keys}(\mathsf{val}\ x = e) = \epsilon$$
$$\mathsf{keys}(\mathsf{p} \oplus_{l,t} L) = (l,t)$$
$$\mathsf{keys}(\&\{(l_i, \mathtt{t}_i, L_i) \Rightarrow P_i\}_{i\in\mathcal{I}}) = [(l_i, \mathtt{t}_i) \mid i \in \mathcal{I}], [\mathsf{keys}(P_i) \mid i \in \mathcal{I}]$$
$$\mathsf{keys}(\mathsf{if}\ e\ \mathsf{then}\ P_1\ \mathsf{else}\ P_2) = \mathsf{keys}(P_1), \mathsf{keys}(P_2)$$
$$\mathsf{keys}(l, t : X(\overline{\mathsf{p}}, \overline{a})) = (l,t)$$
$$\mathsf{keys}(\{P_1\};\ P_2) = \mathsf{keys}(P_1), \mathsf{keys}(P_2)$$

Figure 13 Endpoint projection, continued

2. *(Soundness) If* $\langle\, N,\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{r}} \langle\, N',\ \Sigma',\ K' \,\rangle$ *for some well-formed* N *where* $N \sqsupseteq [\![C]\!]$, *then* $\langle\, C,\ \Sigma,\ K \,\rangle \xrightarrow{\mathsf{p}} \langle\, C',\ \Sigma',\ K' \,\rangle$ *for some* C' *where* $N' \sqsupseteq [\![C']\!]$.

Proof. Relegated to Appendix B. ◀

5 A Non-Blocking Communication API for Choral

Choral [9] is a state-of-the-art choreographic programming language based on Java. Because Choral is designed to interoperate with Java, its syntax differs from that of our formal model: for instance, data locations are lifted to the type level in Choral and communication is expressed using first-class *channels*. However, the core semantics of Choral can still be understood using simplified choreography models like ours.

Choral's intended programming model consists of sequential processes that block to receive messages. However, to improve performance programmers can use Java's `CompletableFuture` API to introduce intraprocess concurrency and out-of-order execution. This breaks the programming model and introduces CIVs (cf. Section 2) that could cause crashes or silent memory corruption. Motivated by our formal model, we developed *Ozone*: an API for Choral programmers to safely mix choreographies with futures. As an added benefit, the Ozone API allows Choral programmers to safely use unreliable transport protocols such as UDP without causing CIVs or hangs. In this section, we introduce Choral and Ozone and we give an illustrative evaluation to compare the two approaches.

5.1 Concurrent Messages

We introduce the Ozone API with an implementation of the choreographic procedure from Figure 2. The implementation is shown in Figure 14, which defines a class called `ConcurrentSend` parameterized by four roles (i.e. process parameters): `KS`, `CS`, `S`, and `C`. In this class, the `start` method implements the procedure itself. As in our formal model, the procedure is parameterized by distributed data: On line 3, parameter `key` is a `String` located at `KS`; `txt` is a `String` located at `CS`; and `client` is a `Client` object at `C`, representing the client's user interface. The `start` procedure is also parameterized by session tokens, which we introduced in Section 2, on line 4. The parameter `Token@(KS, CS, S, C) tok` is syntactic sugar for the parameter *list* `Token@KS tok_KS, ..., Token@C tok_C`.[1] The last

[1] This syntactic sugar is provided for readability and is not currently supported by the Choral compiler. We will also use syntactic sugar for lambda expressions and omit obvious type annotations later in this section. Our actual implementation uses desugared versions of the syntax.

```
1  public class ConcurrentSend@(KS, CS, S, C) {
2      public void start(
3          String@KS key, String@CS txt, Client@C client,
4          Token@(KS, CS, S, C) tok,
5          AsyncChannel@(KS, S) ch1, AsyncChannel@(CS, S) ch2, AsyncChannel@(S, C) ch3
6      ) {
7          // Services send data to the server.
8          CompletableFuture@S keyS = ch1.fcom(key, 1@(KS,S), tok);
9          CompletableFuture@S txtS = ch2.fcom(txt, 2@(CS,S), tok);
10
11         // Server forwards data to the client.
12         ch3.fcom(keyS, 3@(S,C), tok)
13             .thenAccept(client::decrypt);
14         ch3.fcom(txtS, 4@(S,C), tok)
15             .thenAccept(client::display);
16     }
17 }
```

Figure 14 An implementation of the choreography in Figure 2 using Choral and the Ozone API.

three parameters on line 5 are *channels*. In Choral, channels are used to communicate data from one role to another. If `ch` is a channel of type `Channel@(A,B)<T>` and `e` is an expression of type `T@A`, then the expression `ch.com(e)` is a communication that produces a value of type `T@B`.

Our main contribution in the Ozone API is a custom channel `AsyncChannel@(A,B)<T>` with a method `fcom` for safely communicating data with non-blocking semantics. The `fcom` method is similar `com`, but with the following differences:

- Whereas `com` takes one argument, `fcom` takes three: a payload, a line number, and a session token. The latter two arguments form an integrity key, of which both the sender and receiver have a copy.
- When the receiver `B` executes a `com` instruction, its thread becomes blocked until the value (of type `T@B`) has been delivered. In contrast, `fcom` creates a Java *future* (of type `CompletableFuture@B<T>`) which is a placeholder at `B` that will hold a value of type `T` once the message is delivered. Instead of blocking, `fcom` immediately returns that future to the calling thread. The thread can then assign a callback to handle the message and proceed with other useful work.

Lines 8 and 9 of Figure 14 show `fcom` being used to transport `key` and `txt` to the server `S`. The expression `1@(KS, S)` is sugar for the list `1@KS, 1@S` and we assume the replicated value `tok` is expanded into the list `tok_KS, tok_S`. Thus both sender and receiver pass integrity keys as arguments to `fcom`.

Lines 12-15 of Figure 14 show how the server `S` and client `C` use the future values. On line 12, the server uses an overloaded version of `fcom` that takes `CompletableFuture@S` instead of `T@S`. The method assigns to the future a callback, which forwards the result to the client once the future has been completed. The result of `fcom` on line 12 is a `CompletableFuture@C`, to which the client binds a callback on line 13: when the key from `S` finally arrives at `C`, the client will proceed to invoke the method `client.decrypt` with the key as an argument. Lines 14 and 15 do the same, but with the value of `txt`. As we will see below, the values of `key` and `txt` can arrive at the client in any order, so the callbacks on lines 13 and 15 can execute in any order—even in parallel.

```
1  public class ConcurrentSend_KS {
2    public void start(
3      String key, Token tok_KS,
4      AsyncChannel ch1
5    ) {
6      ch1.fcom(key, 1, tok_KS);
7    }
8  }
9  public class ConcurrentSend_S {
10   public void start(
11     Token tok_S, AsyncChannel ch1,
12     AsyncChannel ch2, AsyncChannel ch3
13   ) {
14     CompletableFuture keyS =
15       ch1.fcom(1, tok_S);
16     CompletableFuture txtS =
17       ch2.fcom(2, tok_S);
18
19     ch3.com(keyS, 3, tok_S);
20     ch3.com(txtS, 4, tok_S);
21   }
22 }
```

```
23 public class ConcurrentSend_CS {
24   public void start(
25     String txt, Token tok_CS,
26     AsyncChannel ch2
27   ) {
28     ch2.fcom(txt, 2, tok_CS);
29   }
30 }
31
32 public class ConcurrentSend_C {
33   public void start(
34     Client client, Token tok_C,
35     AsyncChannel ch3
36   ) {
37     ch3.fcom(3, tok_C)
38        .thenAccept(client::decrypt);
39     ch3.fcom(4, tok_C)
40        .thenAccept(client::display);
41   }
42 }
```

Figure 15 Endpoint projection of Figure 14.

5.1.1 Endpoint projection

By running the Choral compiler, `ConcurrentSend@(KS,CS,S,C)` is *projected* to generate four Java classes, shown in Figure 15. Each class implements the behavior of its corresponding role. For example, `ConcurrentSend_KS` implements the behavior of `KS`. Its `start` method is parameterized by: `key`, which corresponds to the `key` in Figure 14; `tok_KS`, the copy of the token `tok` belonging to `KS`; and `ch1`, a channel endpoint that connects `KS` to `S`. Following the reasoning in Figure 12, these behaviors will not exhibit deadlocks or communication integrity errors when composed.

Let us see how integrity keys prevent CIVs in Figure 15. Notice that the Choral instruction `CompletableFuture@S keyS = ch1.fcom(key, 1@(KS,S), tok);` on line 8 of Figure 14 is projected into two instructions:

- `ch1.fcom(key, 1, tok_KS)` at the sender `KS`; and
- `CompletableFuture keyS = ch1.fcom(1, tok_S)` at the receiver `S`.

The former instruction is parameterized by a payload and an integrity key and produces nothing. The latter instruction is parameterized only by an integrity key (with no payload) and produces a future. When `KS` sends `key` to `S`, it combines the payload with integrity key `(1, tok_KS)`. Dually, `S` creates a future that will only be completed when a message with the integrity key `(1, tok_S)` is received. Since `tok_KS` and `tok_S` have the same value, the send- and receive-operations are guaranteed to match.

On lines 14-17 of Figure 15, the server `S` sets listeners for `key` and `txt`. On lines 19-20, `S` schedules the values to be forwarded to `C`; notice that even with FIFO channels, `key` and `txt` may arrive in any order. Consequently, `S` may forward their values to `C` in any order. On lines 37-40 of Figure 15, the client creates futures to hold the values of `key` and `txt` and sets callbacks to be invoked when the values arrive. Here we see the importance of integrity keys: the client uses `(3, tok_C)` and `(4, tok_C)` to disambiguate the `key` message from the `txt` message. Without integrity keys, mixing Choral choreographies with Java Futures

```
1  public class ConcurrentSend2@(KS, CS, S, C) {
2    public void start( ... ) {
3      ...
4      ch3.fcom(keyS, 3@(S,C), tok, 1000@C) // time out after 1000 ms
5          .exceptionally(_ -> null@C)      // default to null on timeout
6          .thenAccept(client::decrypt);
7      ch3.fcom(txtS, 4@(S,C), tok, 1000@C) // time out after 1000 ms
8          .exceptionally(_ -> "Server timed out"@C) // default content
9          .thenAccept(client::display);
10   }
11 }
```

Figure 16 A Choral choreography using the Ozone API to time out when messages take more than a second to arrive.

would be unsafe. As shown in Section 4.3, our solution is correct even when the underlying transport protocol can deliver messages out of order.

5.2 Handling dropped messages

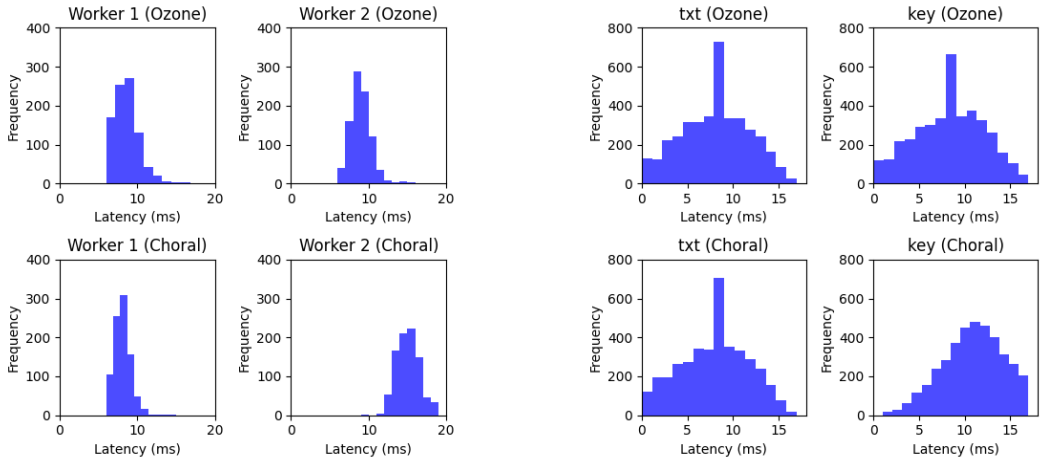
Section 2 introduced communication integrity violations (CIVs). One source of CIVs not considered in that section is *dropped messages*: Figure 16 shows how, if messages are not tagged with unique integrity keys, a dropped message can cause silent data corruption. Because of this scenario, it is unsafe for ordinary Choral programs to simply "time out" when a message has taken too long to arrive. A complete treatment of dropped messages in O_3 would require a significantly more complex model [18]. However, we argue informally that the Ozone API allows programmers to handle dropped messages safely and idiomatically.

Figure 16 shows a modified snippet of `ConcurrentSend` that uses yet another overloaded version of `fcom`. In this version of the method, the receiver process (in this case `C`) passes a non-negative value as a timeout (in this case 1000 milliseconds). When the timeout elapses, a thread on the receiver process will complete the future *exceptionally*, triggering the `.exceptionally` callback (lines 5 and 8) to complete the future with a default value. For example, on lines 4-6, if the message from the server is not received within one second, the client will invoke `decrypt(null)`. Crucially, the callback on lines 6 will only ever be invoked once: If the message arrives after the future timed out, the callback will not be triggered.

One complication of our approach is garbage collection. Recall that in an asynchronous distributed computing model, messages can take any amount of time to be transmitted from sender to receiver. Hence it is theoretically always possible for a message to be delivered after its future has timed out. If the future was immediately garbage collected after timing out, this message would appear "fresh": the receiver would save the message indefinitely, not knowing that its payload is no longer needed, and create a memory leak. In theory, this means the receiver can never garbage collect futures that have timed out—this creates yet another memory leak. However, in practice it may suffice to assume that all messages arrive within a certain deadline T (say, one minute) or not at all. This heuristic allows timed-out futures to be garbage collected after T has elapsed, while avoiding memory leaks in most situations.

```
1  public class ConcurrentSend2_C {
2      public void start( ... ) {
3          ch3.com(3, tok)
4              .exceptionally(_ -> null)
5              .thenAccept(client::decrypt);
6          ch3.com(4, tok)
7              .exceptionally(_ -> "Server timed out")
8              .thenAccept(client::display);
9      }
10 }
```

Figure 17 Endpoint projection of Figure 16 (representative example).



(a) Concurrent producers

(b) Concurrent forwarders

Figure 18 Microbenchmarks.

5.3 Procedure calls

Section 5.1 showed how the line numbers in an integrity key could prevent CIVs. We now briefly show how the *tokens* in an integrity key prevent *interprocedural* CIVs. Figure 19 depicts a choreography that invokes two instances of `ConcurrentSend2`: the first instance with client `C1`, and the second instance with client `C2`. On lines 12 and 16, the roles all compute fresh tokens for each procedure they're involved in, like in our formal model; the syntax `tok.nextToken( 0@(KS,CS,S,C1) )` is sugar for `tok_KS.nextToken(0@KS), ..., tok_C1.nextToken(0@C1)`, and the method `t.nextToken(l)` implements the function $\mathsf{nextToken}(l, t)$. These fresh tokens ensure that, even if messages from `KS` to `S` are delivered out of order (as might occur in the UDP transport protocol) there is no chance that messages from the first procedure invocation will be confused for messages from the second invocation.

5.4 Evaluation

We evaluated Ozone using two microbenchmarks.

The first microbenchmark is a modified version of Figure 1 from the Introduction. We implemented two versions of the microbenchmark: one using Ozone, and the other using

```
public class ConcurrentClients@(KS, CS, S, C1, C2) {
    public void start(
        AsyncChannel@(KS, S) ch1, AsyncChannel@(CS, S) ch2,
        AsyncChannel@(S, C1) ch3, AsyncChannel@(S, C2) ch4,
        KeyService@KS keyService, ContentService@CS contentService,
        Client@C1 client1, String@(KS, CS) clientID1,
        Client@C2 client2, String@(KS, CS) clientID2,
        Token@(KS, CS, S, C1, C2) tok
    ) {
        (new ConcurrentSend2()).start(ch1, ch2, ch3,
            keyService.getKey(clientID1), contentService.getContent(clientID1),
            client1, tok.nextToken( 0@(KS,CS,S,C1) ));

        (new ConcurrentSend2()).start(ch1, ch2, ch4,
            keyService.getKey(clientID2), contentService.getContent(clientID2),
            client2, tok.nextToken( 1@(KS,CS,S,C2) ));
    }
}
```

Figure 19 A Choral choreography invoking `ConcurrentSend2`.

```
public class ConcurrentClients_KS {
  public void start( ... ) {
    (new ConcurrentSend2_KS()).start(ch1,
        keyService.getKey(clientID1),
        tok.next(0));

    (new ConcurrentSend2_KS()).start(ch1,
        keyService.getKey(clientID2),
        tok.next(1));
  }
}
```

```
public class ConcurrentClients_S {
  public void start( ... ) {
    (new ConcurrentSend2_S()).start(
        ch1, ch2, ch3, tok.next(0));

    (new ConcurrentSend2_S()).start(
        ch1, ch2, ch3, tok.next(1));
  }
}
```

Figure 20 Endpoint projection of Figure 19 (representative examples).

Choral's existing API. To measure the effect of network latency, we used network emulation to add 2 milliseconds of latency and 2 milliseconds of normally-distributed jitter. To simulate computation, the server COMPUTE() function sleeps for 5 milliseconds. Figure 18a shows the end-to-end latency experienced by the two workers in each implementation. With the ordinary Choral API, Worker 1 (p_1 in Figure 1) consistently outperforms Worker 2 (p_2) because the server always prioritizes the first worker. With the Ozone API, requests from the two workers are handled fairly, resulting in lower latency for Worker 2. Worker 1 experiences slightly higher latency due to the cases where Worker 2's message arrives while Worker 1's message is still being processed.

The second microbenchmark is a modified version of Figure 2 from Section 2. As before, we compared an Ozone implementation against an ordinary Choral implementation and we inserted delays to simulate a system under load. To measure time accurately, every iteration of the choreography is initiated and terminated by the server. Figure 18b shows the time, measured by the server, for the client (c in Figure 2) to acknowledge receipt of *key* and *txt*. With the Choral API, the average latency for *txt* is higher than *key* because *txt* cannot be forwarded until *key* is received. With the Ozone API, the two values can be forwarded out of order and therefore have identical latency graphs.

6 Related Work

In early choreographic languages, the sequencing operator $I;\ C$ had strict sequential semantics; concurrency could only be introduced via an explicit parallelism operator $C \,||\, C'$ [20, 13, 4]. Explicit parallelism was later replaced by a relaxed sequencing operator $I;\ C$ that would allow instructions in C to be evaluated before I under certain conditions [5]. Our present work makes the sequencing operator even more relaxed, allowing all instructions to be executed out of order, up to data- and control-dependency.

Our present work is closely related to choreographic *multicoms*: sets of communications that can be executed out of order, up to data dependency [8]. However, multicoms do not allow *computation* to be performed out of order, as in Figure 1c. Multicoms therefore do not need to address the communication integrity problem, which we focus on in this work.

To the best of our knowledge, our choreography model is the first to allow non-FIFO communication between processes. We have also observed that the Ozone API can recover from dropped messages by using timeouts, although we do not formalize this. Choreographies with unreliable communication were formalized in the RC model [18]. RC and O_3 both attach tags to messages to prevent CIVs but the former only uses dynamic counters, which are insufficient when processes can execute out of order. On the other hand, RC allows processes to test if sending a message succeeded, which our model does not.

In terms of expressivity, there is significant overlap between our model and *nondeterministic choreographies* [17], which use an explicit *choreographic choice* operator $C +_{\mathsf{p}} C'$. Nondeterministic choreographies can implement the execution in Figure 1c with:

$$\begin{pmatrix} \mathsf{buyer}_1.id \to \mathsf{val\ seller}.id_1; \\ \mathsf{buyer}_2.id \to \mathsf{val\ seller}.id_2; \\ \cdots \end{pmatrix} +_{\mathsf{seller}} \begin{pmatrix} \mathsf{buyer}_2.id \to \mathsf{val\ seller}.id_2; \\ \mathsf{buyer}_1.id \to \mathsf{val\ seller}.id_1; \\ \cdots \end{pmatrix}$$

Figure 2 can also be expressed with nondeterministic choreographies:

$$\begin{pmatrix} 1 : \mathsf{cs}.getText() \to \mathsf{val\ s}.txt; \\ 2 : \mathsf{s} \to \mathsf{c}[\textsc{TxtFirst}]; \\ 3 : \mathsf{s}.txt \to \mathsf{val\ c}.txt; \\ 4 : \mathsf{c}.display(\mathsf{c}.txt); \\ 5 : \mathsf{ks}.getKey() \to \mathsf{val\ s}.key; \\ 6 : \mathsf{s}.key \to \mathsf{val\ c}.key; \\ 7 : \mathsf{c}.decrypt(\mathsf{c}.key) \end{pmatrix} +_{\mathsf{s}} \begin{pmatrix} 8 : \mathsf{ks}.getKey() \to \mathsf{val\ s}.key; \\ 9 : \mathsf{s} \to \mathsf{c}[\textsc{KeyFirst}]; \\ 10 : \mathsf{s}.key \to \mathsf{val\ c}.key; \\ 11 : \mathsf{c}.decrypt(\mathsf{c}.key); \\ 12 : \mathsf{cs}.getText() \to \mathsf{val\ s}.txt; \\ 13 : \mathsf{s}.txt \to \mathsf{val\ c}.txt; \\ 13 : \mathsf{c}.display(\mathsf{c}.txt) \end{pmatrix}$$

Compared to O_3, these implementations are much larger and use selections instead of integrity keys to prevent CIVs. Nondeterministic choreographies are also sensitive to instruction reordering: the seemingly innocuous refactor of moving line 12 up to line 10 would completely destroy the extra concurrency of receiving messages out of order. Thus, our approach is both more robust and much simpler for the programmer.

On the other hand, nondeterministic choreographies can express some nondeterministic programs that our model cannot. For example, choreographic choice can assign different variable names to messages, according to their arrival order. Doing so is a key ingredient in some distributed algorithms, such as Paxos [12]. Other choreographic languages that include nondeterministic operators include those presented in [13] and [3], but they do not support computation (a requirement for choreographic programming) or recursion.

Previous works investigated different ways of modeling asynchronous communication in choreographic languages by making send actions non-blocking [5, 11, 6, 19, 10, 17], but none

of them considered non-blocking reception. Thus, they are not expressive enough to capture the behaviors that we are interested in here, like the one in Figure 1c.

7 Conclusion

We investigated a model for choreographic programming in which processes can execute out of order and messages can be reordered by the network. These features improve the performance of choreographies, without requiring programmers to rewrite their code, by allowing processes to better overlap communication with computation. However, compilers that use these features must have mechanisms in place to prevent communication integrity violations (CIVs). We presented a scheme to prevent CIVs by attaching dynamically-computed integrity keys to each message. Our results enlarge the class of behaviors that can be captured with choreographic programming without renouncing its correctness guarantees.

An important aspect for future work is confluence. Statements can read and write to the local state of a process, so executing statements out of order can cause nondeterminism. Sometimes this nondeterminism is desirable (for instance, to implement consensus algorithms) but sometimes the nondeterminism is unexpected and causes bugs. In our formal model, nondeterminism could be controlled manually by allowing programmers to insert synthetic data dependencies. For example, below we use a hypothetical keyword $\mathsf{barrier}_\mathsf{p}$ to prevent a file from being closed before it has been written-to:

$$\mathsf{val}\ \mathsf{p}.file = open(\text{“foo.txt”});\ \mathsf{p}.write(\mathsf{p}.file, \text{“hello”});\ \mathsf{barrier}_\mathsf{p};\ \mathsf{p}.close(\mathsf{p}.file)$$

More generally, future work could develop a static analysis that identifies when two statements are not safe to execute out of order.

Another opportunity for static analysis to improve on our work concerns the size of session tokens. We chose to represent session tokens as lists of integers, which allowed processes to compute new session tokens without coordinating with one another. However, this encoding means the size of a token is proportional to the depth of the call stack—a problem for tail-recursive programs such as *StreamIt* in Figure 7b. Fortunately, it is easy to see that communication integrity in *StreamIt* could be achieved in constant space by representing the token as a single integer, incremented upon each recursive call—assuming that processes do not participate in multiple instances of the choreography concurrently. With static analysis, a compiler could identify such programs and use a more efficient session token representation.

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A Well-formedness

$$\begin{aligned}
&\mathsf{fv}(0) = \emptyset \\
&\mathsf{fv}(\{\, C \,\}; C') = \mathsf{fv}(C) \cup \mathsf{fv}(C') \\
&\mathsf{fv}(l, t : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x;\ C') = \mathsf{fv}(e) \cup \mathsf{fv}(C') \setminus \{\mathsf{q}.x\} \\
&\mathsf{fv}(l, t : \mathsf{val}\ \mathsf{p}.x = e;\ C') = \mathsf{fv}(e) \cup \mathsf{fv}(C') \setminus \{\mathsf{p}.x\} \\
&\mathsf{fv}(l, t : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2;\ C') = \\
&\quad \mathsf{fv}(e) \cup \mathsf{fv}(C_1) \cup \mathsf{fv}(C_2) \cup \mathsf{fv}(C') \\
&\mathsf{fv}(l, t : X(\overline{\mathsf{p}}, \overline{a});\ C') = \{\mathsf{p}.x \mid \mathsf{p}.x \in \overline{a}\} \cup \mathsf{fv}(C') \\
&\mathsf{fv}(l, t : \overline{\mathsf{p}}.\, X(\overline{\mathsf{q}}, \overline{a})\, \{\, C \,\}) = \{\mathsf{p}.x \mid \mathsf{p}.x \in \overline{a}\} \cup \mathsf{fv}(C') \\
&\mathsf{fv}(I;\ C) = \mathsf{fv}(C) \text{ otherwise.}
\end{aligned}$$

$$\begin{aligned}
&\mathsf{pn}(0) = \emptyset \\
&\mathsf{pn}(I;\ C) = \mathsf{pn}(I) \cup \mathsf{pn}(C) \\
&\mathsf{pn}(\{\, C \,\}) = \mathsf{pn}(C) \\
&\mathsf{pn}(l, t : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x) = \{\mathsf{p}, \mathsf{q}\} \\
&\mathsf{pn}(l, t : \mathsf{p} \rightsquigarrow \mathsf{q}.x) = \{\mathsf{q}\} \\
&\mathsf{pn}(l, t : \mathsf{p} \to \mathsf{q}[L]) = \{\mathsf{p}, \mathsf{q}\} \\
&\mathsf{pn}(l, t : \mathsf{p} \rightsquigarrow \mathsf{q}[L]) = \{\mathsf{q}\} \\
&\mathsf{pn}(l, t : \mathsf{val}\ \mathsf{p}.x = e) = \{\mathsf{p}\} \\
&\mathsf{pn}(l, t : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2) = \\
&\quad \{\mathsf{p}\} \cup \mathsf{pn}(C_1) \cup \mathsf{pn}(C_2) \\
&\mathsf{pn}(l, t : X(\overline{\mathsf{p}}, \overline{a})) = \overline{\mathsf{p}} \\
&\mathsf{pn}(l, t : \overline{\mathsf{q}}.\, X(\overline{\mathsf{p}}, \overline{a})\, \{\, C \,\}) = \overline{\mathsf{p}} \\
&\mathsf{pn}(v@\mathsf{p}) = \{\mathsf{p}\} \\
&\mathsf{pn}(\mathsf{p}.x) = \{\mathsf{p}\}
\end{aligned}$$

$$\begin{aligned}
&\mathsf{stats}(0) = \epsilon \\
&\mathsf{stats}(I;\ C) = \mathsf{stats}(I),\ \mathsf{stats}(C) \\
&\mathsf{stats}(\{\, C \,\}) = \mathsf{stats}(C) \\
&\mathsf{stats}(l, t : \mathsf{if}\ e@\mathsf{q}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2) = (l, t : \mathsf{if}\ e@\mathsf{q}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2),\ \mathsf{stats}(C_1),\ \mathsf{stats}(C_2) \\
&\mathsf{stats}(l, t : \overline{\mathsf{q}}.\, X(\overline{\mathsf{p}}, \overline{a})\, \{\, C \,\}) = (l, t : \overline{\mathsf{q}}.\, X(\overline{\mathsf{p}}, \overline{a})\, \{\, C \,\}), \mathsf{stats}(C) \\
&\mathsf{stats}(l, t : \eta) = (l, t : \eta) \text{ otherwise} \\
&\mathsf{stats}(C) = [\mathsf{stats}(C) \mid \mathsf{p} \in \mathsf{pn}(C)] \\
&\mathsf{keys}(C) = [(l, t) \mid (l, t : \eta) \in \mathsf{stats}(C)] \\
&\mathsf{keys}_{\mathsf{q}}(C) = [(l, t) \mid (l, t : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x) \in \mathsf{stats}(C)], [(l, t) \mid (l, t : \mathsf{p} \rightsquigarrow \mathsf{q}.x) \in \mathsf{stats}(C)]
\end{aligned}$$

$$\frac{\begin{array}{c}\mathsf{pn}(C) \subseteq \mathsf{dom}(\Sigma) \qquad \mathsf{pn}(C) \subseteq \mathsf{dom}(K) \qquad \mathsf{fv}(C) = \emptyset \\ \mathsf{keys}(C) \text{ distinct} \qquad \forall (l, t) \in \mathsf{keys}(C),\ t \neq \mathsf{t} \\ \forall I_1, I_2 \in \mathsf{stats}(C), \text{ if } \mathsf{key}(I_1) \prec \mathsf{key}(I_2) \text{ then } I_1 = l_1, t_1 : \overline{\mathsf{q}}.X(\overline{\mathsf{p}}, \overline{a})\{C'\} \text{ and } I_2 \in \mathsf{stats}(C') \\ (X(\overline{\mathsf{p}}, \overline{\mathsf{p}.x}) = C)\checkmark \text{ for each } (X(\overline{\mathsf{p}}, \overline{\mathsf{p}.x}) = C) \in \mathscr{C} \qquad \langle I, K\rangle \checkmark \text{ for each } I \in \mathsf{stats}(C)\end{array}}{\langle \mathscr{C}, C, \Sigma, K\rangle \checkmark}\ \text{C-WF}$$

$$\frac{\begin{array}{c}\overline{\mathsf{p}} \text{ distinct} \qquad \overline{\mathsf{p}.x} \text{ distinct} \qquad \mathsf{pn}(C) \subseteq \overline{\mathsf{p}} \\ \forall \mathsf{p}.x \in \overline{\mathsf{p}.x},\ \mathsf{p} \in \overline{\mathsf{p}} \qquad \langle I, K\rangle \checkmark \text{ for each } I \in \mathsf{stats}(C) \\ C \text{ contains no runtime terms} \qquad \mathsf{keys}(C) \text{ distinct} \qquad \forall (l, t) \in \mathsf{keys}(C),\ t = \mathsf{t}\end{array}}{X(\overline{\mathsf{p}}, \overline{\mathsf{p}.x}) = C \checkmark}\ \text{C-WF-Def}$$

$$\frac{\forall v,\ (l, \mathsf{t}_i, v) \notin K(\mathsf{q})}{\langle l, \mathsf{t}_i : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x,\ K\rangle \checkmark}\ \text{C-WF-Send}$$

$$\frac{\exists! v,\ (l, \mathsf{t}_i, v) \in K(\mathsf{q})}{\langle l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x,\ K\rangle \checkmark}\ \text{C-WF-Recv}$$

$$\frac{(l, \mathsf{t}_i, L) \notin K(\mathsf{q})}{\langle l, \mathsf{t}_i : \mathsf{p} \to \mathsf{q}[L],\ K\rangle \checkmark}\ \text{C-WF-Select}$$

$$\frac{(l, \mathsf{t}_i, L) \in K(\mathsf{q})}{\langle l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}[L],\ K\rangle \checkmark}\ \text{C-WF-OnSelect}$$

$$\frac{}{\langle l, \mathsf{t}_i : \mathsf{val}\ \mathsf{p}.x = e,\ K\rangle \checkmark}\ \text{C-WF-Compute}$$

$$\frac{C_1, C_2 \text{ contain no runtime terms}}{\langle l, \mathsf{t}_i : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2,\ K\rangle \checkmark}\ \text{C-WF-If}$$

$$\frac{}{\langle \{\, C_1 \,\},\ K\rangle \checkmark}\ \text{C-WF-Block}$$

$$\frac{\begin{array}{c}(X(\mathsf{q}_1, \ldots, \mathsf{q}_n, \mathsf{q}^1.x_1, \ldots, q^m.x_m) = C) \in \mathscr{C} \\ \mathsf{p}_1, \ldots, \mathsf{p}_n \text{ distinct} \qquad \forall i \le n, j \le m, \text{ if } \mathsf{pn}(a_j) = \mathsf{p}_i \text{ then } \mathsf{q}^j = \mathsf{q}_i\end{array}}{\langle l, \mathsf{t}_i : X(\mathsf{p}_1, \ldots, \mathsf{p}_n, a_1, \ldots, a_m),\ K\rangle \checkmark}\ \text{C-WF-Call}$$

$$\frac{\begin{array}{c}\langle l, \mathsf{t}_i : X(\overline{\mathsf{q}}, \overline{a}),\ K\rangle \checkmark \qquad (X(\mathsf{q}_1, \ldots, \mathsf{q}_n, \mathsf{q}^1.x_1, \ldots, q^m.x_m) = C') \in \mathscr{C} \\ \{\mathsf{r}_1, \ldots, \mathsf{r}_k\} \subseteq \{\mathsf{p}_1, \ldots, \mathsf{p}_n\} \qquad \forall i \le k, j \le n \text{ if } \mathsf{r}_i = \mathsf{p}_j \text{ then } [\![C]\!]_{\mathsf{r}_i} = [\![C']\!]_{\mathsf{q}_j}\end{array}}{\langle l, \mathsf{t}_i : \mathsf{r}_1, \ldots, \mathsf{r}_k.\, X(\mathsf{p}_1, \ldots, \mathsf{p}_n, a_1, \ldots, a_m)\,\{\, C \,\},\ K\rangle \checkmark}\ \text{C-WF-Calling}$$

B EPP Theorem

▶ **Theorem 6** (EPP Theorem). *Let* $\langle C,\ \Sigma,\ K \rangle$ *be a well-formed configuration.*

1. *(Completeness) If* $\langle C,\ \Sigma,\ K \rangle \xrightarrow{\mathsf{p}} \langle C',\ \Sigma',\ K' \rangle$ *then* $\langle [\![C]\!],\ \Sigma,\ K \rangle \xrightarrow{\mathsf{p}} \langle N',\ \Sigma',\ K' \rangle$ *for some well-formed* N' *where* $N' \sqsupseteq [\![C']\!]$.
2. *(Soundness) If* $\langle N,\ \Sigma,\ K \rangle \xrightarrow{\mathsf{r}} \langle N',\ \Sigma',\ K' \rangle$ *for some well-formed* N *where* $N \sqsupseteq [\![C]\!]$, *then* $\langle C,\ \Sigma,\ K \rangle \xrightarrow{\mathsf{p}} \langle C',\ \Sigma',\ K' \rangle$ *for some* C' *where* $N' \sqsupseteq [\![C']\!]$.

Completeness, sketch. The proof proceeds by induction on the derivation $\mathcal{D}$ that produces $\langle C,\ \Sigma,\ K \rangle \xrightarrow{\mathsf{p}} \langle C',\ \Sigma',\ K' \rangle$. In most cases it suffices to let $N' = [\![C']\!]$. In the case of C-If, we find a network N' such that $N' \neq [\![C']\!]$ but $N' \sqsupseteq [\![C']\!]$.
Because O_3 uses scoped variables, the proof requires the substitution lemmas in Lemma 5. The C-Delay rule is also slightly novel: if I is not a selection at q then completeness requires Lemma 5 (4) and an application of P-Delay. ◀

Soundness, sketch. The proof proceeds by induction on the structure of the choreography C. The base case, $C \equiv 0$, is trivial. Otherwise, $C \equiv (I;\ C')$ and there is a distinct case for each instruction I. The key novelty in this proof is handling P-Delay and the frequent use of well-formedness to guarantee that K does not contain certain messages. We consider two representative cases.

Case 1. Let $C = l, \mathsf{t}_i : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x;\ C''$. Then, by the definition of EPP, N has the form

$$N = \mathsf{p}[\mathsf{q}\ !_{l,\mathsf{t}_i}\ e;\ P] \mid \mathsf{q}[?_{l,\mathsf{t}_i}\ x;\ Q] \mid (N \setminus \mathsf{p}, \mathsf{q}).$$

There are three sub-cases.

Case 1.1. Assume $\mathsf{r} = \mathsf{p}$. In standard choreography models, this case can only proceed by P-Send. In our model, it could also proceed by P-Delay. Hence there are two sub-cases:
Case 1.1.1. (P-Send) Satisfied by letting $C' = l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x;\ C''$.
Case 1.1.2. (P-Delay) By the induction hypothesis, there exists C''' such that $N''' \sqsupseteq [\![C''']\!]$. The case is then satisfied by letting $C' = l, \mathsf{t}_i : \mathsf{p}.e \to \mathsf{val}\ \mathsf{q}.x;\ C'''$.

Case 1.2. Assume $\mathsf{r} = \mathsf{q}$. Again, we consider the two rules by which the case could proceed:
Case 1.2.1. (P-Recv) We must show that it is impossible for q to receive a message in $\langle N,\ \Sigma,\ K \rangle$. Since C is well-formed, it cannot contain a communication-in-progress term $l, \mathsf{t}_i : \mathsf{p} \rightsquigarrow \mathsf{q}.x$ with the integrity key (l, t_i). Hence M does not contain any messages of the form (l, t_i, v).

Case 1.2.2. (P-Delay) This case proceeds similarly to the previous P-Delay case.

Case 1.3. Assume $\mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\}$. Follows from the induction hypothesis, as in Case 1.1.2.

Case 2. Let $C = l, \mathsf{t}_i : \mathsf{if}\ e@\mathsf{p}\ \mathsf{then}\ C_1\ \mathsf{else}\ C_2;\ C_3$. Then N has the form

$$N = \mathsf{p}[\mathsf{if}\ e\ \mathsf{then}\ P_1\ \mathsf{else}\ P_2;\ P_3]\ |\ \left(\prod_{q_i \in \overline{\mathsf{q}}} \mathsf{q}_i[\&\{(l_j, \mathsf{t}_j, L_j) \Rightarrow Q_{i,j}\}_{j \in \mathcal{I}};\ Q_i]\right)\ |\ (N \setminus \mathsf{p}, \overline{\mathsf{q}}).$$

Consider the case where $\mathsf{r} = \mathsf{q}_i \in \overline{\mathsf{q}}$ and $\left\langle N,\ \Sigma,\ K \right\rangle \xrightarrow{\mathsf{q}} \left\langle N',\ \Sigma',\ K' \right\rangle$ proceeds by P-OnSelect. That is,

$$\mathsf{q}_i[\&\{(l_j, \mathsf{t}_j, L_j) \Rightarrow Q_{i,j}\}_{j \in \mathcal{I}};\ Q_i] \xrightarrow{\mathsf{q}_i} \mathsf{q}_i[Q_{i,k};\ Q_i]$$

for some $k \in \mathcal{J}$. We must show that this case is impossible. Notice that the step can only occur if $(l, \mathsf{t}_k, L_k) \in K(\mathsf{q}_i)$. Such a message can only occur if $l, \mathsf{t}_k : \mathsf{p} \rightsquigarrow \mathsf{q}[L_k]$ occurs in C_1, C_2, or C_3. Because C is well-formed, C_1 and C_2 do not contain runtime terms; hence the term could only occur in C_3. But then Q_i would contain a branch $(l, \mathsf{t}_k, L_k) \Rightarrow Q'$ and $\mathsf{keys}(N|_{\mathsf{q}_i})$ would not be distinct; a contradiction of Lemma 5 (5). ◀